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Graviton loops and brane observables

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Abstract: We discuss how to consistently perform effective Lagrangian computations in quantum gravity with branes in compact extra dimensions. A reparametrization invariant and infrared finite result is obtained in a non trivial way. It is crucial to properly account for brane fluctuations and to correctly identify physical observables. Our results correct some confusing claims in the literature. We discuss the implications of graviton loops on electroweak precision observables and on the muon $g-2$ in models with large extra dimensions. We model the leading effects, not controlled by effective field theory, by introducing a hard momentum cut-off.

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1. Introduction

No known experimental constraint firmly excludes the possibility that Kaluza Klein (KK) excitations of the graviton propagating in $\delta \geq 2$ large extra dimensions will affect future particle physics experiments [1, 2].[1] After removing all non propagating degrees of freedom by a suitable choice of coordinates, many authors computed the signals of KK graviton emission at tree level [4, 5, 6]. Some authors also considered 1-loop effects [6]–[10]: since they affect observables measured with higher precision, they can compete with tree level effects. The result was not the expected one. Consider for example the graviton correction to the Higgs mass. At first sight one would estimate it as

$$\delta m_h^2 \sim \sum_n \int_k \frac{(m_h k/M_4)^2}{(k^2+m_n^2)(k^2+m_h^2)} \sim \sum_n \frac{m_h^2 \Lambda^2}{(4\pi)^2 M_4^2} \sim m_h^2 \Big(\frac{\Lambda}{M_D}\Big)^{2+\delta}, \tag{1.1}$$

where M_D, M_4 are the gravitational scales of the D-dimensional and 4-dimensional theory respectively, and $\Lambda \sim M_D$ parameterizes the unknown quantum gravity ultraviolet cutoff. At a closer look [6]–[9] the effect seems to be much larger. To understand that, consider the propagator for the physical $J = 2$ nth KK graviton with mass m_n and 4-momentum k_μ [4, 6]

$$G^{(n)}_{\mu\nu} G^{(m)}_{\rho\sigma} = \frac{i\delta_{n,-m}}{(k^2-m_n^2)} \frac{1}{2} \left(t_{\mu\rho}t_{\nu\sigma} + t_{\mu\sigma}t_{\nu\rho} - \frac{2}{3} t_{\mu\nu}t_{\rho\sigma} \right), \tag{1.2}$$

where $t_{\mu\nu} \equiv \eta_{\mu\nu} - k_\mu k_\nu/m_n^2$. If the terms enhanced by powers of k/m_n were to fully contribute to quantum corrections, the k factors would give a highly ultraviolet (UV) divergent loop effect. More importantly, when $\delta < 4$ the $1/m_n$ factors would also give a strong infrared (IR) enhancement of the sum over KK modes. At the end the correction would be a factor $(M_D R)^{4-\delta}$ larger than the naive one in eq. (1.1), where R is the size of the extra dimensions. This kind of behavior, indeed observed in [6]–[9], would exclude the possibility that $\delta < 4$ large extra dimensions (i.e. $R \gg 1/M_D$) solve the hierarchy problem.

The above argument on the fate of the k/m_n terms must however be wrong, and for a very simple reason. Indeed one could choose to fix the D-dimensional reparametrization invariance by the de Donder gauge choice, in which the graviton propagator contains no k/m_n terms. This is in complete analogy with the case of a massive vector boson, where the propagator contains k/m_n terms in the unitary gauge, while no such term is present in the Feynman gauge. Therefore k/m_n terms cannot affect gauge-invariant physical observables. This suggests that there must be something missing or incorrect in the computations so far performed.

The purpose of the present paper is to devise all the elements that are needed for a fully consistent computation. The guideline is to respect the full D-dimensional

[1] The case $\delta = 2$ is excluded by bounds on emission of KK modes with a small mass $\lesssim 100$ MeV in supernovae [1, 3], if the extra dimensions are flat. In principle, one could save collider signals (due to heavy KK modes) by assuming that the compact dimensions are curved on length scales $\gtrsim (100\mathrm{MeV})^{-1}$, so that the light KK are lifted.

general coordinate covariance. First of all it is crucial to fix the gauge by the Faddeev-Popov procedure and to choose a covariant regulator. If the regulation of the loop integral is not performed with the due care, spurious UV and IR divergences can appear. Secondly, one has to remember that the position of the brane depends on the system of coordinates, and therefore brane fluctuations (*branons*) must be taken into account in order to respect general covariance. Finally one must carefully identify which are the physical observables in the presence of gravity: misidentification of the true observables can yield spurious gauge dependence and IR divergences.

One of our results is that all the puzzling effects found in the existing literature cancel out in a fully consistent calculation when one computes physical observables. For example the Higgs mass term and the oblique S, T, U parameters [11] are not physical observables (except in particular cases). So they receive gauge dependent quantum gravity corrections, which in some cases are even enhanced by powers of RM_D. These infrared pathologies, which would invalidate perturbation theory (for instance $RM_D \sim 10^{15}$ if $R \sim$ mm and $M_D \sim$ TeV), are absent in the corrections that affect the corresponding physical observables, the pole higgs mass and the $\epsilon_1, \epsilon_2, \epsilon_3$ parameters [12].

In our study we treat quantum gravity and the brane by the method of effective field theory (EFT) [13, 14]. We do so in the absence of a realistic fundamental description of the SM on a brane.[2] The effective Lagrangian summarizes all our low energy knowledge of gravitational interactions with SM particles. By our method we could perform a fully consistent computation of the 1-loop quantum gravity corrections to electroweak precision observables. However the dominant effects are strictly speaking uncalculable, as they are saturated in the UV where we loose control of the theory. We can only parameterize these effects in terms of a UV cutoff Λ.[3] The calculable piece is the one saturated in the infrared, but this is only of order $(M_Z/M_D)^{2+\delta}$. Therefore, introducing a UV cut-off Λ, we will only compute a particular combination of observables, which is affected by just a few simple Feynman diagrams. For the full set of observables we will limit ourselves to a qualitative discussion.

While the discussion of the phenomenology is somewhat limited by the powerlike UV divergences, we stress that the main goal of the present paper is conceptual. In this respect the most important (and new) result is that brane motions have to be properly taken into account. In order to understand this issue better we have also

[2] Interesting attempts based on D-brane intersections [15] give 'semi-realistic' models with extra charged matter with respect to the SM. The stability of these configurations is an open question.

[3] A string model could provide a physical realization of this cut off. However at the level of the present model building technology there are many free parameters specifying the moduli and the brane configuration [15, 16, 17]. Therefore, even if we were able to reproduce the SM, the predictive power on quantum corrections (for example on the muon $g-2$) would probably be limited. Of course it would still be important to have one such model. Indeed it would also be interesting to have a field theoretic brane model in the spirit of [18, 19].

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considered the case of a brane living at an orbifold fixed point, for which the branons are projected out. In this case gauge independence of observables is met through tadpole diagrams specific of orbifold compactifications, rather than by branon loops. The technology developed in this paper may prove useful in future work. One possible application is the brane to brane mediation of supersymmetry breaking through bulk gravity at 1-loop. This effect is computable and represents the leading correction to anomaly mediated soft terms: depending on its sign it may cure the tachyon problem of anomaly mediation.

The paper is structured as follows. In section 2 we discuss our lagrangian and the effective field theory philosophy. We also introduce various gauge fixing conditions for the gravitational field and explain the rôle of the branons. In section 3 we calculate the corrections to the masses of scalars and vectors, on and off the brane. We explain what gauge independence means in quantum gravity, and show that physical quantities are gauge independent. We also give the example of a brane at an orbifold fixed point, for which branons are not needed. In section 4 we derive experimental bounds on low energy quantum gravity from precision measurements and from the anomalous magnetic moment of the μ. In section 5 we summarize. Finally in the appendices we describe how to derive graviton-matter vertices and collect our results for the corrections to brane observables.

2. Effective lagrangians for gravitons and branes

2.1 Pure gravity

We study gravity in $\mathbb{R}^d \times \mathcal{M}$ where the extra dimension $\mathcal{M}$ is a compact manifold of dimension δ. Not knowing which manifold is of physical interest (if any), we consider the simplest one: a δ-torus T^δ with a single radius R and volume $V = (2\pi R)^\delta$. We perturbatively expand the classical Einstein-Hilbert action around the flat metric $g_{MN} = \eta_{MN} + \kappa\, h_{MN}$, $\eta_{MN} = (+1, -1, -1, -1, \ldots)$ in terms of the graviton field h_{MN}:

$$\begin{aligned} \mathcal{S} &= \frac{\bar{M}_D^{D-2}}{2} \int d^D X \sqrt{g}\, R \qquad (2.1) \\ &= \frac{1}{2} \int d^D X \left(- h^{MN} \Box h_{MN} + h \Box h - 2h^{MN} \partial_M \partial_N h + 2h^{MR} \partial_R \partial_S h^S_{\ M} \right) + \mathcal{O}(\kappa)\,, \end{aligned}$$

where $D = d + \delta$, $h \equiv h^M_M$ and we used η_{MN} to raise and lower indices. We use upper (lower) case latin letters for D-dimensional (extra-dimensional) indices and greek letters for d-dimensional indices; in particular we decompose the D-dimensional coordinates as $X^M = (x^\mu, y^i)$. We do not fix $d = 4$ since we will use dimensional regularization. Following the notation of ref. [4] we have defined

$$\kappa^2 \equiv 4 \bar{M}_D^{2-D}\,, \qquad \bar{M}_d^{d-2} = V \bar{M}_D^{D-2} = R^\delta M_D^{D-2} \qquad (2.2)$$

$\bar{M}_d$ is the effective reduced Planck mass as measured by a d-dimensional observer, $\bar{M}_D$ is the corresponding parameter in D dimension and M_D is defined by (2.2). With this convention the equations of motion read:

$$R_{MN} - \frac{1}{2} g_{MN} R = -\frac{1}{\bar{M}_D^{D-2}} T_{MN} \,. \tag{2.3}$$

Before inverting the quadratic term in eq. (2.1) to obtain the propagators, one must fix the reparametrization invariance; we follow the Faddeev-Popov procedure and introduce a set of ξ-gauges by adding to the lagrangian the gauge-fixing term $\mathcal{L}_{\rm GF} = -F^2/\xi$, where

$$F_N = \partial^\mu \left(h_{\mu N} - \frac{1}{2} \eta_{\mu N} h \right) + \xi \partial^i \left(h_{iN} - \frac{1}{2\xi} \eta_{iN} h \right) . \tag{2.4}$$

This particular choice breaks the D-dimensional Lorentz symmetry of the flat background metric for generic values of ξ and interpolates between the usual de Donder and unitary gauge, obtained respectively in the limit $\xi \to 1, \infty$. The functional integral gets multiplied by the Faddeev-Popov determinant, exponentiated in the usual way by introducing 'ghost' fields η_M, $\bar{\eta}_M$:

$$\mathcal{L}_{\rm ghost} = \int d^D X \, d^D X' \; \bar{\eta}_N(X) \frac{\delta F_N(X)}{\delta \lambda_M(X')} \bigg|_{\lambda=0} \eta_M(X') \,, \tag{2.5}$$

where λ is the gauge parameter for reparametrizations.

The kinetic term for the graviton field is a (messy) 3×3 matrix which mixes tensor $h_{\mu\nu}$, vector $h_{\mu i}$ and scalar h_{ij} modes. Since interactions are more easily written in terms of the $h_{\mu\nu}$, $h_{\mu i}$ and h_{ij} components of the D-dimensional graviton field h_{MN}, it is more convenient to write the propagator in this basis rather than in the gauge-dependent mass eigenstate basis. For example matter fields confined on a straight d-dimensional brane at leading order couple only to the tensorial $h_{\mu\nu}$ mode.

By decomposing the graviton field h_{MN} in its Fourier harmonics

$$h_{MN}(x, y) = \frac{1}{\sqrt{V}} \sum_{n \in \mathbb{Z}^\delta} h_{MN}^{(n)}(x) \, e^{in \cdot y/R} \tag{2.6}$$

and integrating the Einstein-Hilbert lagrangian over the extra-coordinates, one obtains the d-dimensional lagrangian for KK modes.

Notice that $\partial^i h_{iN}$ can be interpreted as Goldstone bosons eaten in a gravitational Higgs mechanism to form massive tensors and vectors. We are classifying particles by the d-dimensional Poincarè group. By this interpretation, eq. (2.4) is the analogue of 't Hooft's ξ gauge in spontaneously broken gauge theories. For $\xi \to \infty$ we get the unitary gauge [4, 6] in which only the physical degrees of freedom propagate. In this limit the ξ gauge propagator for the modes $h_{MN} = (h_{\mu\nu},\, h_{ij},\, h_{\mu j})$ simplifies to

$$\overbrace{h^{(n)}_{MN} h^{(n')}_{M'N'}} = \frac{1}{2}\frac{i\delta_{n,-n'}}{(k^2 - m_n^2)} \times \tag{2.7}$$
$$\times \begin{pmatrix} t_{\mu\mu'}t_{\nu\nu'} + t_{\mu\nu'}t_{\mu'\nu} - \frac{2}{D-2}t_{\mu\nu}t_{\mu'\nu'} & \frac{2}{D-2}P_{i'j'}t_{\mu\nu} & 0 \\ \frac{2}{D-2}P_{ij}t_{\mu'\nu'} & P_{ii'}P_{jj'} + P_{ij'}P_{ji'} - \frac{2}{D-2}P_{ij}P_{i'j'} & 0 \\ 0 & 0 & -P_{ii'}t_{\mu\mu'} \end{pmatrix}$$

where k is the d-dimensional momentum,

$$P_{ij} \equiv \delta_{ij} - \frac{n_i n_j}{n^2}\,, \qquad t_{\mu\nu} \equiv \eta_{\mu\nu} - \frac{k_\mu k_\nu}{m_n^2} \tag{2.8}$$

and $m_n^2 = n^2/R^2$ is the mass squared for the nth KK excitation, having defined $n^2 \equiv -n_i n_j \eta^{ij} = n_i n_j \delta_{ij}$. In appendix A we derive this propagator by working in the unitary gauge with physical fields. As usual 'Goldstone' bosons and 'ghosts' get infinitely massive when $\xi \to \infty$ but they do not decouple: loop corrections computed in the unitary gauge ($\xi = \infty$) by propagating only the physical fields are different from the limit $\xi \to \infty$ of loop effects computed in a ξ gauge [20]. Of course the mismatch disappears in physical quantities.

For $\xi = 1$ we get instead the de Donder gauge, where the propagator has the covariant form:

$$\overbrace{h_{MN} h_{M'N'}} = \frac{i}{2K^2}\left(\eta_{MM'}\eta_{NN'} + \eta_{MN'}\eta_{NM'} - \frac{2}{D-2}\eta_{MN}\eta_{M'N'}\right), \tag{2.9}$$

where K is the D-dimensional momentum. In matrix notation, for the single KK mode:

$$\overbrace{h^{(n)}_{MN} h^{(n')}_{M'N'}} = \frac{1}{2}\frac{i\delta_{n,-n'}}{(k^2 - m_n^2)} \times \tag{2.10}$$
$$\times \begin{pmatrix} \eta_{\mu\mu'}\eta_{\nu\nu'} + \eta_{\mu\nu'}\eta_{\nu\mu'} - \frac{2}{D-2}\eta_{\mu\nu}\eta_{\mu'\nu'} & \frac{2}{D-2}\delta_{i'j'}\eta_{\mu\nu} & 0 \\ \frac{2}{D-2}\delta_{ij}\eta_{\mu'\nu'} & \delta_{ii'}\delta_{jj'} + \delta_{ij'}\delta_{ji'} - \frac{2}{D-2}\delta_{ij}\delta_{i'j'} & 0 \\ 0 & 0 & -\delta_{ii'}\eta_{\mu\mu'} \end{pmatrix}.$$

We have thus shown that the propagator in the de Donder and unitary gauges has the same form up to longitudinal k/m_n terms. For compactness, we do not write explicitly the propagator in a generic ξ-gauge.

2.2 Gravity and branes

We want to study quantum gravity corrections to the physical observables of a field theory living on a d-dimensional brane in a δ-dimensional compact space $\mathcal{M}$. At the end we will identify the brane theory with the Standard Model. The gravitational lagrangian has been discussed in the previous subsection. We now discuss the brane lagrangian.

We use an effective field theory (EFT) approach where the fundamental description of the particles and of the brane is not specified [14]. In the regime of validity of EFT, the particles are treated as point-like and the brane is treated as infinitely thin in the extra dimensions. This requires a little explanation. If ρ is the brane true transverse size, our EFT is only valid at energy scales $\ll 1/\rho$. The brane also generally has a finite tension $\tau \equiv f^d$. This gives rise to a gravitational field behaving like $f^d/M_D^{d+\delta-2}r^{\delta-2} \equiv (r_G/r)^{\delta-2}$ at a distance r far away from the brane in the extra space. We focus on $\delta > 2$ (for $\delta = 2$ the background is locally flat with a conical singularity at the brane position). The gravitational radius r_G controls the distance at which the geometry is curved. One can then think of different possibilities for the brane structure. If $\rho > r_G$ the brane is similar to a big star where the geometry nowhere strongly deviates from the flat, and $1/\rho$ truly represents the UV cut-off of our EFT. On the other hand for $\rho \ll r_G$ it is the gravitational radius that sets the UV cut-off. Physics at energies $> 1/r_G$ would probe the gravitational structure of the brane, which is non-universal and model dependent. One example is a black brane where at $r \sim r_G$ a black-hole horizon is present. Another different example is given by the solution studied in [21], where there is no horizon and a naked singularity is avoided by a finite brane size. In the latter case the coupling of bulk gravitons to the brane is dramatically changed at energies $> 1/r_G$. As we are only interested in universal features we will assume that the UV cut off $\Lambda_{\rm UV}$ that limits the use of our EFT is bounded by $\min(1/\rho, 1/r_G)$. In the regime of validity of EFT we can treat the background metric as approximately flat and treat the effects of the brane tension as perturbations. Notice indeed that at energy E the latter are controlled by the small parameter $E^{\delta-2}f^d/M_D^{d+\delta-2} \equiv (Er_G)^{\delta-2} < (\Lambda_{\rm UV}r_G)^{\delta-2} \ll 1$.

Two possibilities are given: either the brane can freely move in the bulk or sit at a fixed point, if the compact space $\mathcal{M}$ has any. Let us consider the former case first. The immersion of the brane in the D-dimensional space is parameterized by D functions $X^M(z)$, where z^μ are the d local coordinates on the brane. The brane action must be invariant under both D-dimensional coordinate changes (under which X^M transform and z^μ are unchanged) and under reparametrizations of the brane coordinates z^μ. An invariant brane action can be built using the induced metric

$$g^{\rm ind}_{\mu\nu}(z) = \frac{\partial X^M(z)}{\partial z^\mu}\frac{\partial X^N(z)}{\partial z^\nu} g_{MN}(X(z))\,. \tag{2.11}$$

Since $g^{\rm ind}_{\mu\nu}$ is a scalar under D-dimensional reparametrizations, we only need to respect brane reparametrizations by the use of $g^{\rm ind}_{\mu\nu}$ itself. The description of the brane position by the $X^M(z)$ is of course redundant. We can eliminate this redundancy by using the remaining gauge freedom represented by brane reparametrizations. We stress that we cannot use D-dimensional diffeomorphisms for which the gauge has been completely fixed in the previous section. A convenient choice of brane coordinates is $x^\mu = z^\mu$, $y^i = \xi^i(x^\mu)$. This choice completely fixes brane reparametrizations without the need of introducing additional ghost fields (the ghost determinant is trivial) [14]. We call the ξ^i *branons*.

As we said the branons cannot be thrown away because we have already completely fixed the D-dimensional reparametrization gauge invariance. However in the previous section one could have chosen a different class of coordinate gauges, one in which the brane always sits at a given point in $\mathcal{M}$. This different choice would explicitly break translation invariance in the extra dimensions. What becomes of the branons in these different gauges? They are still there but as longitudinal modes of a combination of graviphotons: the branons can indeed be interpreted as the Goldstone bosons of broken translation invariance in the extra dimensions [14]. We find it more convenient to gauge fix the graviton in the more standard way and keep the branons. Notice that, consistently with their Goldstone character, in the limit in which gravity decouples ($M_D \to \infty$) the branons survive. Their physical effects can therefore be studied independently of gravity [22]. Quantum fluctuations of the branons are controlled by $1/\tau$ (the analogue of $1/f_\pi^2$ for pions) and become non-perturbative at an energy $E > \sqrt{4\pi} f$ ($E > 4\pi f_\pi$ for pions). Therefore the tension τ sets another sure upper bound on the regime of applicability of EFT.

In terms of the branons ξ^i the induced metric is

$$g_{\mu\nu}^{\rm ind} = g_{\mu\nu} - g_{\mu i} g_{\nu j} g^{ij} + (D_\mu \xi_i)(D_\nu \xi_j) g^{ij} \equiv \eta_{\mu\nu} + \tilde{h}_{\mu\nu}\,, \tag{2.12}$$

where $D_\mu \xi_i \equiv \partial_\mu \xi_i + g_{\mu i}$ and the metric g_{MN} is evaluated at the brane location $y^i = \xi^i$. For 1-loop computations we need $\tilde{h}_{\mu\nu}$ up to quadratic order in ξ

$$\tilde{h}_{\mu\nu} = \kappa h_{\mu\nu} + (\partial_\mu \xi_i)(\partial_\nu \xi^i) + \kappa(\xi^i \partial_i h_{\mu\nu} + h_{i\mu}\partial_\nu \xi^i + h_{i\nu}\partial_\mu \xi^i) + \cdots\,, \tag{2.13}$$

where now h is the graviton field evaluated at the brane rest position $y^i = 0$. The brane lagrangian is given by

$$\mathcal{S}_{\rm brane} = \int d^4x \left[-\tau \sqrt{\det g_{\mu\nu}^{\rm ind}} + \mathcal{L}_{\rm SM} + \cdots \right], \tag{2.14}$$

where $\mathcal{L}_{\rm SM}$ is the covariant brane lagrangian (that we will identify with the SM lagrangian), while the dots indicate all terms involving higher derivatives, the Riemann tensor for the induced metric [23] or the extrinsic curvature. By expanding the tension term up to quadratic order in the branons and gravitons we get

$$\mathcal{L}_{\rm mix} = -\frac{\tau}{2}\left[(\partial_\mu \xi_i)(\partial^\mu \xi^i) + \kappa(h_\mu^\mu + \xi^i \partial_i h_\mu^\mu + 2 h_{i\mu}\partial^\mu \xi^i) + \kappa^2 B^{\mu\nu\rho\sigma} h_{\mu\nu} h_{\rho\sigma} \right] + \cdots \tag{2.15}$$

which shows a mixing between ξ and h (see appendix B for the definition of the tensor B). As we will discuss shortly, in order to consistently compute virtual graviton effects this mixing has to be taken into account. Notice also that there is a linear term in h, since the massive brane is a source of gravity. We will comment below about when and how can this term be neglected. The interaction of gravitons and branons with SM fields is encoded in the covariant dependence of $\mathcal{L}_{\rm SM}$ on the induced metric. At quadratic order we have

$$\mathcal{L}_{\rm SM} = L_{\rm SM} + \kappa L^{\mu\nu}\tilde{h}_{\mu\nu} + \kappa^2 L^{\mu\nu\mu'\nu'}\tilde{h}_{\mu\nu}\tilde{h}_{\mu'\nu} + \cdots \qquad (T_{\mu\nu} \equiv -2L_{\mu\nu}), \tag{2.16}$$

where the explicit formulae are given in appendix B.

2.3 Gravity and vector bosons

In section 2.1 we have described the gauge fixing procedure for a theory of pure gravity. If gauge fields A_M are present, the lagrangian has both internal and gravitational gauge invariance, which can be fixed through a delta functional $\delta(F(h, A))$

$$F(h, A) = [f_1(h, A), f_2(h, A)] \tag{2.17}$$

in the functional integral imposing $f_1(A, h) = 0$, $f_2(A, h) = 0$. This is equivalent to adding the gauge fixing term $\mathcal{L}_{\rm GF} = -f_1^2/\xi - f_2^2/2\zeta$ in the lagrangian and the Faddeev-Popov determinant in the functional integral

$$\det \frac{\delta F(h, A)}{\delta\lambda} = \det \begin{pmatrix} \delta f_1/\delta\lambda_1 & \delta f_1/\delta\lambda_2 \\ \delta f_2/\delta\lambda_1 & \delta f_2/\delta\lambda_2 \end{pmatrix}, \tag{2.18}$$

where λ_1, λ_2 are the gauge parameters for diffeomorphisms and internal gauge transformations respectively. $\delta f_2/\delta\lambda_1$ is generically non zero because vector bosons are 'charged' under gravity. However the graviton field is neutral under charge transformations, so that for a reasonable gauge-fixing function f_1 which doesn't involve the vector bosons (in particular for f_1 as in eq. (2.4)) the determinant factorizes:

$$\det \frac{\delta F}{\delta\lambda} = \det \frac{\delta f_1}{\delta\lambda_1} \cdot \det \frac{\delta f_2}{\delta\lambda_2} \tag{2.19}$$

and the two factors can be exponentiated separately in the usual way. Notice that it is convenient to choose a non covariant gauge-fixing f_2 for the photons in order to avoid additional couplings with gravitons. (A non-covariant f_2 should not cause any panic: reparametrizations are already broken by the gravitational gauge fixing f_1). We have explicitly checked that the simple gauge fixing

$$\mathcal{L}_{\rm GF} = -\frac{1}{2\zeta}(\partial_M A_N \eta^{MN})^2 \tag{2.20}$$

gives the same results as other more involved choices.

In a theory with vectors that acquire mass M_v through the Higgs mechanism, as in the Standard Model, the gauge-fixing term will contain the Goldstone bosons ϕ_G field as well. Even with the simple gauge fixing

$$\mathcal{L}_{\rm GF} = -\frac{1}{2\zeta}\left(\partial_M A_N \eta^{MN} - M_v\zeta\phi_G\right)^2 \tag{2.21}$$

there is a cubic vector-Goldstone-graviton interaction: in a generic metric the gauge fixing does not fully cancel the kinetic mixing between the Goldstones and the vector ($M_v A_M \partial_N \phi_G g^{MN}\sqrt{g}$) present in the lagrangian. Such gauge fixing can be easily adapted to vector bosons confined on a brane.

2.4 Gravity and fermions

Finally, we sketch how to extend our analysis to the important case of fermions. It is well known that GL(D) does not admit spinor representations and in order to deal with fermions, we need some extra structure: the vierbein $E^A_{\ M}$ and its inverse $E_A^{\ M}$ defined by

$$g_{MN} = \eta_{AB}\, E^A_{\ M} E^B_{\ N} \qquad E^A_{\ M} E_B^{\ M} = \delta^A_B\,. \tag{2.22}$$

Where capital letters from the beginning of the latin alphabet $A, B, C, \ldots$ denote D-dimensional Lorentz indices. The vierbein basis definition introduces an additional gauge symmetry, besides diffeomorphisms, due to the freedom in (2.22) to rotate E acting with a local SO($D-1,1$) transformation. In absence of torsion, the compatibility condition between the metric and the connection ω, allows to express the latter in terms of the vierbein E. Then, once the vierbein is defined, the introduction of spinors is rather straightforward (see for instance [24]), a collection of the relevant formulae can be found in appendix B. Around a flat background we can parametrize the vierbein as $E^A_{\ M} = \delta^A_M + \kappa B^A_{\ M}$. In terms of the quantum field $B^A_{\ M}$ the metric fluctuation is then

$$h_{MN} = B_{MN} + B_{NM} + \kappa\, B^O_{\ \ M} B_{ON}\,, \tag{2.23}$$

where $B_{MN} = \eta_{MA} B^A_{\ N}$ and similarly all indices are raised and lowered by the Minkowski metric η_{AB}. The gravitational action, when expressed in terms of E (or equivalently in terms of B), is invariant under the infinitesimal local Lorentz transformation

$$\delta B^A_{\ M} = \kappa^{-1} \Omega^A_B(X)\, \left(\delta^A_M + \kappa B^B_{\ M}\right)\,, \qquad \Omega_{AB}(X) = -\Omega_{BA}(X)\,. \tag{2.24}$$

A convenient gauge choice is [25]

$$B_{MN} - B_{NM} = 0\,, \tag{2.25}$$

The great advantage of (2.25) is that Lorentz ghosts are absent [25] and that it makes possible the elimination of the vierbein fields, order by order in κ, in favor of the quantum metric h [26]. Indeed, in the gauge (2.25) one can easily express B in terms of h by solving (2.23)

$$B_{MN} = \frac{1}{2} h_{MN} - \frac{1}{8}\kappa\, h^A_{\ \ M} h_{AN} + \mathrm{O}(\kappa^2)\,. \tag{2.26}$$

As a result, even when fermions are present, at the perturbative level, the quantum fluctuations of the geometry are encoded in h and our formalism can be applied without modifications.

A similar procedure can be applied to fermions living on a $(d-1)$-brane. These are spinors of SO($d-1,1$) and in order to write an invariant lagrangian one needs the induced vierbein on the brane, which is now a $d \times d$ matrix $e^a_{\ \mu}$. In what follows we indicate by lower-case latin letters $a, b, c, \ldots$ the d-dimensional Lorentz indices. In ref. [14] it was shown how to construct $e^a{}_\mu$ out of E^A_M and of the brane immersion $X^M(z)$. Basically it has the form

$$e^a{}_\mu(z) = R^a_A E^A_M \left(X(z)\right) \partial_\mu X^M(z)\,, \tag{2.27}$$

where R^a_A is a SO$(D-1,1)$ rotation matrix which depends on E^A_M and $X^M(z)$. Under a SO$(D-1,1)$ rotation $E^A_M \to \Omega(X)^A_B E^B_M$, the induced vierbein undergoes a SO$(d-1,1)$ rotation $e^a{}_\mu \to \omega(z)^a_b e^b{}_\mu$. By fixing the brane reprametrizations keeping just the branons (as done in the previous section) and by fixing D-dimensional Lorentz transformations as shown in this section, $e^a{}_\mu$ is written as a function of $\xi^i(x)$ and h_{MN}. However it is a fairly complicated expression. Calculations can be simplified by using the local Lorentz symmetry $e^a{}_\mu \to \omega^a_b(x) e^b{}_\mu$ to rotate the induced vierbein to a more convenient form (fermions rotate $\psi^\alpha \to \omega^\alpha_\beta \psi^\beta$ by the spinorial representation ω^α_β). Precisely as we did with E^A_M it is useful to rotate $e^a{}_\mu$ to a symmetric matrix

$$e^a{}_\mu = \delta^a_\mu + b^a{}_\mu \qquad \eta_{\nu a} b^a{}_\mu = \eta_{\mu a} b^a{}_\nu \tag{2.28}$$

from which by using $g^{\rm ind}_{\mu\nu} = e^a{}_\mu e^{a\nu}$ and eq. (2.12) we obtain the analogue of eq. (2.26)

$$b_{\mu\nu} = \frac{1}{2}\tilde{h}_{\mu\nu} - \frac{1}{8}\tilde{h}^\rho{}_\mu \tilde{h}_{\rho\nu} + O(\tilde{h}^3)\,. \tag{2.29}$$

3. Loop corrections to brane observables

We now have all the ingredients to perform some illustrative computations. We will focus on the one loop correction to the masses of scalars and vectors living on the brane.

In order to do a meaningful computation we must employ a regularization that respects D-dimensional reparametrization invariance. The result will depend on the choice of the regulator. The simplest thing could be cutting the loop integrals at Λ. Since this is not an invariant regulator, we would get a meaningless result that also depends on the choice of the loop integration momentum. A better possibility consists in dividing all graviton propagators by some power of $(1 - p^2/\Lambda^2)$. It is possible to obtain this Pauli-Villars (PV) regulator in a covariant way by adding to the action suitable higher derivative reparametrization invariant terms involving just the metric. We will instead employ the standard extension of dimensional regularization to the case in which both continuum and discrete momentum are involved (see the appendices for details). Of course by this method we are only sensitive to the "physical" logarithmic divergences, while all power divergences are automatically removed. Nonetheless from our results it will be clear that by choosing a regulator sensitive to power divergences (like PV) for the sum over KK we would still not have the ΛR terms of refs. [6]–[9] in physical quantities.

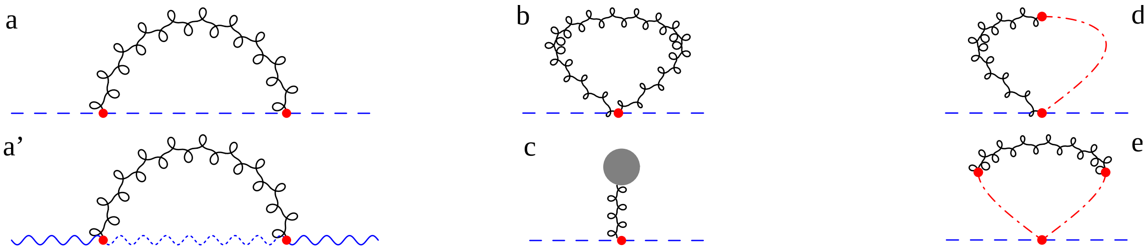

Figure 1: One-loop gravitational corrections to the pole mass of a scalar on a brane from gravitons (diagrams a,b,c) and from graviton/branon mixing (diagrams d,e) at zeroth order in the brane tension. The mass of a vector particle on a brane also gets corrections from vector-Goldstone-graviton vertices (diagram a$'$). Gravitons (branons, scalars, vectors, Goldstones) are drawn as pig-tail (dot-dashed, dashed, wavy, dashed-wavy) lines.

3.1 Brane in a torus

Consider now the one-loop graviton correction to the *pole* mass m_0 of a minimally coupled scalar living on a *straight* brane located at the point $y^i = 0$ of a torus T^δ. As we explained in section 2.2, in the regime of validity of EFT ($E < 1/r_G$) the brane tension can be treated as a perturbation in the gravitational dynamics. Therefore it makes sense to expand the corrections to our observables in a power series in τ. Let us focus on the lowest order effects, i.e. those that go like τ^0.[4] The diagrams that contribute at order τ^0 are shown in figure 1. Notice that diagrams (d) and (e) also involve branons: in these diagrams the τ^{-1} from branon propagation is compensated by the τ^1 in the graviton-branon mixing insertion. Notice also that the tadpole diagram (c) gives no contribution. Due to momentum conservation in the extra dimensions (valid at zeroth order in τ) only the zero modes mediate this tadpole, these are the 4d graviton and the radion. Whatever mechanism stabilizes the radion giving also a vanishing effective 4d cosmological constant generates a tadpole that cancels (c) exactly at the minimum of the radion potential. Of course exact cancellation of the 4d cosmological constant requires the usual fine tuning. Now, the genuine graviton diagrams (a) and (b) give a correction

$$\delta m_0^2(\mathrm{a}+\mathrm{b}) = \frac{i}{\bar{M}_d^{d-2}} \sum_n \int_k \langle \phi(-p)| - 2L^{\mu\nu}L^{\rho\sigma} + 4iL^{\mu\nu\rho\sigma}|\phi(p)\rangle \left(\overline{h_{\mu\nu}^{(n)} h_{\rho\sigma}^{(-n)}}\right), \qquad (3.1)$$

where $p^2 = m_0^2$ is the squared momentum of the on-shell scalars (m_0 is the tree level mass).

[4]Notice that there are also corrections from pure branon exchange, which go like inverse powers of τ and which persist when gravity is turned off. The lowest, physically meaningful correction of this type to the scalar mass comes at two loops and goes like $\delta m_0/m_0 \sim m_0^8/\tau^2$. The $1/\tau$ effects can be bigger than the gravitational ones we study, but they are physically independent [22]. Thus it makes sense to focus only on the latter.

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The branon contributions (d) and (e) give

$$\delta m_0^2(\mathrm{d}+\mathrm{e}) = \frac{i}{\bar{M}_d^{d-2}}\langle\phi(-p)|T_{\mu\mu}|\phi(p)\rangle \sum_n \int_k F_n(k)\,, \tag{3.2}$$

where $\langle\phi(-p)|T_{\mu\mu}|\phi(p)\rangle = 2m_0^2$ and $F_n(k)$ represents the contribution of the loop in figure 1d,e. The sum of all contributions in the interpolating ξ gauge defined in section 2 gives

$$\begin{aligned}\delta m_0^2 = \frac{m_0^2}{32\bar{M}_d^{d-2}\pi^2 d(d+\delta-2)}\sum_n \Big\{ & f(\xi,d,\delta)A_0(m_n^2) - d(d-2)(\delta-4)A_0(m_0^2) + \\ & + 4d[(d-2)m_n^2 - 2(d+\delta-3)m_0^2]\times \\ & \times B_0(m_0^2,m_n^2,m_0^2) + \\ & + d\delta(d-2)m_n^2 B_1(m_0^2,m_n^2,m_0^2)\Big\}\end{aligned} \tag{3.3}$$

$$\begin{aligned} f(\xi,d,\delta) = {} & 4(2\xi-1)^{d/2-1}[2-\delta(2\xi-1)(\xi-1)-\xi(10-3d+3\xi(d-4))] + \\ & +2\xi^{d/2}[3d\delta + d\xi(\delta-2)+d^2(\xi+1)+4-12\xi+2\delta(\xi+\delta-4)] + \\ & +2(d-2)[-\delta+2d(d+\delta-2)]\,, \end{aligned} \tag{3.4}$$

where the Passarino-Veltman functions A_0, B_0, B_1 are defined in appendix D, where we describe how the cutoff-independent contribution can be extracted. The mass correction is multiplicative as expected for a minimally coupled scalar: for vanishing tree level mass the scalar is derivatively coupled to gravity. Although in this expression all the terms enhanced by $1/m_n^4$ found in [6]–[9] cancel out mode by mode, we do *not* obtain a gauge-independent result. However the ξ dependent term in $\delta m_0^2/m_0^2$, only depends on M_D and R (and the UV cut-off Λ_{UV} if dimensional regularization is abandoned) but not on m_0^2 itself. So it looks like a universal effect. Indeed one finds the same gauge dependent piece in the correction to the mass of a vector particle. In short the correction to the pole mass of a spin $s=\{0,1\}$ particle on the brane can be written as

$$\delta m_s^2 = 2m_s^2 G(M_D R) + m_s^2\Delta_s(m_s,\bar{\mu},R,M_D)\,, \tag{3.5}$$

where G is the only gauge dependent factor, while Δ_0, Δ_1 are gauge-independent ($\bar{\mu}$ is the renormalization scale). Explicit expressions for these functions can be found in appendix C. Similarly we find that for a localized photon the gravitational correction to the electric charge e has a gauge-dependent factor equal to $[e]G$, where $[e]=2-d/2$ denotes the dimension of the electric charge in d dimensions. The moral of these results is that the gauge dependence can be reabsorbed by changing the normalization of the graviton field g_{MN}. In more physical terms, gauge dependent terms amount to a change of the mass unit: all the dimensionless quantities that we have computed (like m_0/m_1) are gauge independent. In the presence of gravity only dimensionless quantities are real observables, as they are invariant under rescaling of the metric.[5] As a simple further check we have also computed the corrections to the masses of bulk particles. In particular we have focused on the $n = 0$ modes of fields with spin $s = \{0, 1\}$ and bulk mass $\{m'_0, m'_1\}$. Again, we obtain

$$\delta m'^2_s = 2m'^2_s G + m'^2_s \Delta'_s\,, \tag{3.6}$$

where the gauge dependent part is the same as for brane modes but the physically meaningful piece Δ'_s is, as expected, different. Notice that bulk particles do not couple directly to branons, so that there is no analogue of diagrams (d) and (e) for them. On the other hand the bulk modes couple directly to the $h_{i\mu}$ and h_{ij} pieces of the graviton field, which was not the case for the brane modes. In view of these differences, the fact that the gauge depended piece is always the same is a rather non trivial check.

A concluding remark on the $1/m_n^4$ terms found in [6]–[9] is in order. These terms come only from diagram (a), so that the branons play no role in the cancellation of these effects in physical quantities. Furthermore, it is clear that terms of this type could not be physical, as they cannot arise in the $\xi = 1$ gauge. However in gauge dependent quantities they can appear. In the appendix we give the expression of the scalar self-energy at the off-shell point $p_{\rm ext} = 0$, where these unphysical effects are indeed present. Notice that if they appeared in physical quantities there would really be an enhancement of the result by some power of the radius R (IR divergences cannot be thrown away!).

So far we have only considered observables that do not depend at tree level on the size R of the compact extra dimension. A gauge invariant result is obtained in a slightly more complicated way when one considers observables like $m_0/M_{\rm Pl}$ or the ratio between pole masses of different KK excitations. The reason is that R itself is gauge dependent: a discussion of this issue, including a geometrical explanation of this statement, is presented in subsection 3.2.

3.2 Gauge independence of physics and geometry

In this section we want to extend our discussion of gauge invariance to generic observables that depend on the size R of the extra dimensions. To be concrete we will compare two gauge choices, unitary (U) and de Donder (DD). The compact manifold is assumed to be a δ-torus.

The previous results could be restated as follows: in order to get the same physics in the U and DD gauges, all the tree level mass parameters m_U and m_{DD} in the two gauges should be related by

$$m_U^2 = m_{DD}^2[1 + [m^2](G_U - G_{DD})] = m_{DD}^2\lambda_1 \qquad [m^2] = 2\,, \tag{3.7}$$

[5]The gauge dependence of pole masses in quantum gravity was already found and discussed in ref. [27]. In the next section we will give a simple geometrical explanation.

where the G's are the universal quantities given in appendix C. Similar relations hold for parameters with different mass dimensions. This is equivalent to taking as background metrics $\lambda_1\eta_{MN}$ and η_{MN} in respectively the U and DD gauge, but keeping the same tree level mass parameters (i.e. m_{DD}). This is easily seen because the tree level Klein-Gordon operator in U gauge is $\lambda_1^{-1}\eta^{MN}\partial_M\partial_N + m_{DD}^2$.

This argument is basically correct, but not completely. The point is that since the space is not isotropic (there are δ compact directions) the metric rescaling factor λ does not have to be the same for all directions. Then, compatibly with the symmetries of the system, we expect in general the backgrounds

$$g_{MN}^{DD} = \begin{pmatrix} \eta_{\mu\nu} & 0 \\ 0 & \eta_{ij} \end{pmatrix} \qquad g_{MN}^{U} = \begin{pmatrix} \lambda_1\eta_{\mu\nu} & 0 \\ 0 & \lambda_2\eta_{ij} \end{pmatrix} \tag{3.8}$$

with $\lambda_1 \neq \lambda_2$. These relative backgrounds have to be chosen in order to get the same results in the two gauges. Notice that we keep the same periodicity $y_i \sim y_i + 2\pi R$ on the torus. Then, at tree level, the proper length of the period of the torus is rescaled by a factor $\sqrt{\lambda_2}$ in the unitary gauge. In the same gauge the mass shell condition for the mode $\{n_i\}$ is

$$\left(\eta^{\mu\nu}\partial_\mu\partial_\nu + \frac{\lambda_1}{\lambda_2}\frac{n^2}{R^2} + \lambda_1 m^2\right)\phi_n = 0 \tag{3.9}$$

so that R as defined through KK masses is rescaled by $\sqrt{\lambda_2/\lambda_1}$ at tree level, and not by $\sqrt{\lambda_2}$. Finally the tree level d-dimensional Newton constant $G_N = 1/(M_D^{d+\delta-2}R^\delta)$ in the U gauge is given by $G_N^U = G_N\lambda_2^{-\delta/2}\lambda_1^{1-d/2}$. By writing $\lambda_i = 1 + c_i$, at lowest order in the c_i we then have the tree level relations

$$\begin{aligned} m_U &= m_{DD}\left(1 + \frac{c_1}{2}\right) \\ R_U &= R_{DD}\left(1 + \frac{c_2}{2} - \frac{c_1}{2}\right) \\ (G_N)_U &= (G_N)_{DD}\left(1 + \left(1 - \frac{d}{2}\right)c_1 - \frac{\delta}{2}c_2\right), \end{aligned} \tag{3.10}$$

where the radius is here defined through the KK masses. In our calculations so far we only considered the masses of brane modes or bulk zero modes, which do not depend on the radius at tree level. This is why one universal rescaling λ_1 was enough to eliminate spurious gauge effects. By considering the quantum corrections to KK masses or to the d-dimensional Newton constant one finds extra gauge dependence. However we have checked by explicit calculations that it can all be eliminated consistently with eq. (3.10). The corrections to KK masses represent just a direct generalization of the computation of the previous section. On the other hand, the Newton constant requires to compute also the correction to the graviton-matter vertex and to the graviton propagator. Few relevant Feynman diagrams are shown in

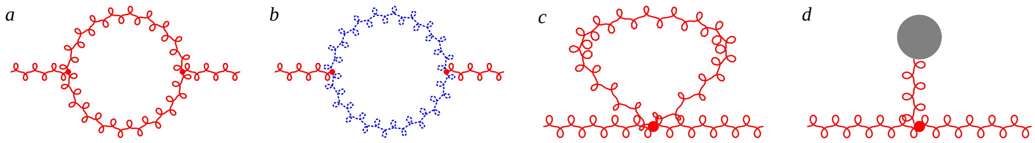

Figure 2: The corrections to the Planck mass is obtained by combining corrections to the graviton propagator (we show the relative Feynman diagrams. Diagram *b* contains a ghost loop) with corrections to the graviton/matter vertex and with corrections to the matter propagator.

fig. 2. This is a lengthy computation[6] upon which the gauge dependence in eq. (3.10) is a non-trivial check. At one loop, we find

$$\lambda_1 = 1 + \frac{d^2 + d(2\delta - 1) + \delta(\delta - 3)}{8\pi^2 \bar{M}_d^{d-2} d(d+\delta-2)} \sum_n A_0(m_n^2)$$
$$\lambda_2 = 1 + \frac{d}{8\pi^2 \bar{M}_d^{d-2}} \sum_n A_0(m_n^2)\,. \qquad (3.11)$$

It is interesting to re-derive the quantities λ_1, λ_2 in a purely geometrical way. For instance λ_2 is fixed by the coordinate independent proper period of the torus. We can easily show this for the case $\delta = 1$, $d = 4$ (the latter choice being made just to simplify the notation). In order to do so we must (arbitrarily) pick a path around the compact dimension, and make sure that working in different gauges the path is kept unchanged. It is convenient to simply pick the path $\mathcal{P}$ defined in unitary coordinates by $x^\mu = 0, y^5 = \tau$ with τ going from 0 to $2\pi R$. Actually any path in the *family* $x^\mu = \text{const}, y^5 = \tau$ ($\tau = [0, 2\pi R]$) would give the same result as it is equivalent by translation invariance of the background.[7] Notice also that in a general coordinate choice, x^μ is not constant along $\mathcal{P}$. In an arbitrary coordinate system the definition $X^M(\tau)$ of $\mathcal{P}$ depends also on the metric itself. It is easy to work out this dependence. The quantity

$$L = \left\langle \int_{\mathcal{P}} \sqrt{g_{MN}\dot{X}^M\dot{X}^N} d\tau \right\rangle \qquad (3.12)$$

must be gauge independent, being the expectation value of a gauge invariant operator. Comparing the calculation of L in the U and DD gauges, we get at 1-loop

[6]For example in the unitary gauge the diagram 2a is obtained by summing 2.588.740 terms. All computations in this work have been done with Mathematica [28].

[7]This is an important point since by construction the unitary coordinate frame, defined by the request that g_{55} and $g_{\mu 5}$ be independent of y^5, is truly a *family* of gauges. This is because the unitary form of the metric is preserved by the "zero mode" coordinate changes $x^\mu \to x^\mu + \epsilon^\mu(x)$, $y^5 \to y^5 + \epsilon^5(x)$ (corresponding to 4-dimensional diffeomorfisms and to the circle isometry). Then since our path $\mathcal{P}$ is defined in a family of gauges it truly designates a family of paths. It is manifest that this family corresponds to the paths related to $\mathcal{P}$ by translation in x. They all have the same length.

order

$$
\begin{aligned}
L &= 2\pi R + \frac{c_2}{2} - \frac{\kappa^2}{8}\langle h_{55}^{(0)} h_{55}^{(0)}\rangle_U \\
&= 2\pi R - \frac{\kappa^2}{8}\langle h_{55}^{(0)} h_{55}^{(0)}\rangle_{DD} + \frac{\kappa^2}{2}\sum_{n\neq 0}\Big[\eta^{\mu\nu}\langle h_{5\mu}^{(n)} h_{5\nu}^{(-n)}\rangle_{DD} + \\
&\qquad\qquad + \frac{1}{4}\Big(\frac{\eta^{\mu\nu}\langle \partial_\mu h_{55}^{(n)} \partial_\nu h_{55}^{(-n)}\rangle_{DD}}{m_n^2} - \langle h_{55}^{(n)} h_{55}^{(-n)}\rangle_{DD}\Big)\Big].
\end{aligned}
$$

In the unitary gauge only the scalar zero mode (radion) h_{55} contributes to L, while in the DD gauge extra contributions from KK graviphotons and graviscalars show up (the latter is zero in dimensional regularization). However in the U gauge there is the tree level term c_2. Notice that in both gauges the gravitational field h is defined to have no tadpoles. This equation fixes c_2 and the result agrees with what found for the physical parameters.

Finally we can fix c_1 by considering the volume element in the two gauges

$$
\Big\langle \int_{T^\delta} \sqrt{g}\; d^\delta y\; d^d x \Big\rangle_U = \Big\langle \int_{T^\delta} \sqrt{g}\; d^\delta y\; d^d x \Big\rangle_{DD}, \tag{3.13}
$$

where we have fully integrated on the torus T^δ. This equality ensures that the non-compact coordinates x represent the same physical distance in the two gauges. The mass of a particle, as defined by the x dependence of the propagator, has then to be the same in the two gauges. Taking the background into account, eq. (3.13) reads at 1-loop

$$
(2\pi R)^\delta \left[1 + \frac{\delta c_2}{2} + \frac{d c_1}{2} + \langle \hat{O}\rangle_U\right] = (2\pi R)^\delta \left[1 + \langle \hat{O}\rangle_{DD}\right], \tag{3.14}
$$

where

$$
\hat{O} = \frac{\kappa^2}{8}\sum_n (\eta^{MN}\eta^{RS} - 2\eta^{MR}\eta^{NS}) h_{MN}^{(n)} h_{RS}^{(-n)}. \tag{3.15}
$$

Using the previous result for c_2 we determine c_1 in agreement with eq. (3.11).

3.3 Higher orders in the brane tension

We have only studied the terms of zeroth order in the tension τ, but things should work out in a similar way order by order in τ. These higher order effects come not only from branon insertion in the diagrams of figure 1a and 1b but also from extra tadpoles. Indeed at order τ there is already at the tree level the tadpole of figure 3.

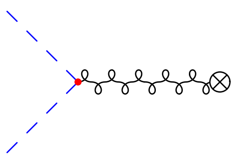

Figure 3: A diagram of order τ^1.

It corresponds to the brane self gravitational field. Of course if we treat the brane as a thin object this field is infinite at the brane itself. This is a UV divergence which is eliminated by adding the suitable counterterms (amounting to a renormalization of the unit length on the brane). Applying our regulator (see appendix D) we get from figure 3, for $d = 4$

$$\delta m_0^2 = m_0^2 \frac{\mu^4}{M_4^2 R^\delta} \frac{8(\delta-2)}{(\delta+2)} \sum_n \frac{1}{m_n^2} = m_0^2 \frac{\mu^4}{M_D^{2+\delta} R^{\delta-2}} \frac{8(\delta-2)}{(\delta+2)} \mathcal{I}_1 \,, \tag{3.16}$$

where $\mathcal{I}_1$ is a constant defined in appendix D. The R dependence is insensitive to the UV cut-off: it measures the deformation of the brane self field due to the finite volume, so it is a well defined quantity in the EFT approach. Notice also that fig. (3) is a brane-to-brane exchange of a bulk graviton like those considered in various phenomenological studies [4, 29]. At one loop figure 3 is dressed into extra tadpole diagrams: we expect that inclusion of these tadpoles will be essential to get gauge independent results at linear order in τ.

3.4 Brane in an orbifold

In section 2.2 we explained that, for a brane living on a smooth space, the branons have to be kept in order to preserve general covariance. Then we have explicitly shown that the branons are needed to restore reparametrization gauge independence of quantum gravity corrections. In this section we show an example of how things work for a brane stuck at a fixed point of an orbifold. Now the brane cannot move, i.e. there is no branon degree of freedom. But at the same time the group of diffeomorfisms is also changed. Indeed when dealing with fixed points it is even superfluous to talk about a brane: at these points we can localize degrees of freedom and interactions respecting the orbifold reparametrization invariance. For simplicity we will consider the simplest case of a brane in $\mathbb{R}^d \times S^1/\mathbb{Z}_2$. The space $S^1/\mathbb{Z}_2$ is a line segment, obtained identifying points in a circle of radius R according to the $\mathbb{Z}_2$ reflection:

$$y \sim 2\pi R - y \,; \qquad y \in [0,\, 2\pi R] \tag{3.17}$$

which has $0, \pi R$ as fixed points. The invariance of the line element ds^2 under $\mathbb{Z}_2$ implies that under the orbifold reflection the metric components $h_{\mu\nu}$, h_{ij} and the ghost field η_μ are even, while $h_{\mu i}$ and η_i are odd. A generic field $f(x,y)$ can be Fourier decomposed according to its parity:

$$\begin{aligned} f(x,y) &= \sum_{n=0}^{+\infty} f^{(n)}(x)\, \Psi_n(y), \qquad & \Psi_n(y) &= \begin{cases} a_n \cos(ny/R) & \text{even} \\ b_n \sin(ny/R) & \text{odd} \end{cases} \\ a_0 &= \frac{1}{\sqrt{2\pi R}}, \quad b_0 = 0 & a_n &= b_n = \frac{1}{\sqrt{\pi R}} \quad n \neq 0 \,. \end{aligned} \tag{3.18}$$

Odd fields do not have a zero mode. We can use the same gauge fixing for reparametrization invariance as before.

The group of diffeomorfisms on the orbifold is defined by the transformations $x^\mu \to f^\mu(x,y)$, $y \to f^5(x,y)$ with f^μ even and f^5 odd under orbifold reflection (both

f^μ and f^5 have period $2\pi R$). Notice that the boundaries $y = 0$ and $y = \pi R$ are left fixed. A brane at $y = 0$ remains a brane at $y = 0$ in all reference frames. Even if we do not let the brane fluctuate we still obtain consistent results. On the other hand for a brane at a generic $y \neq 0, \pi R$, its position depends on the reference frame and we are forced to let it fluctuate. What is special about the fixed points is that we have thrown away enough gauge degrees of freedom ($g_{\mu 5}(y = 0, \pi R) = 0$) that we can live without branons. Let us see this explicitly.

The computation of the gravitational corrections in the orbifold geometry is similar to the previous ones, but with some important differences. Consider the case of a brane sitting at a generic point y. Contrary to the circle case, the y dependence in the coupling between matter on the brane and gravity does not cancel out in physical amplitudes; for instance, the cross section for the production of an individual KK graviton mode is proportional to $\cos^2(ny/R)$. It is not a surprise that this factor depends on y: $S^1/\mathbb{Z}_2$ is not an homogeneous space. Similarly, the branon contribution in graviton loops gets multiplied by a factor $\sin^2(ny/R)$, showing that their presence is not necessary when $y = 0$. However, having altered by a y-dependent factor the relative weight between graviton and branon effects, we apparently no longer get a gauge invariant result. We now show that we must take into account a new type of graviton tadpoles that were absent on a homogenous space.

The conservation law of fifth dimensional momentum is altered since some of the harmonics are projected out by the $\mathbb{Z}_2$ symmetry. In a vertex with three lines carrying momenta $n_i \geq 0$, $i = 1, 2, 3$ along the fifth dimension it reads $n_1 \pm n_2 \pm n_3 = 0$. The propagator of matter on the brane is corrected by new tadpole diagrams, like (1c), but with non zero extra-dimensional momentum $2n$ on the tadpole graviton line. The blob in figure $1c$ can be either a graviton loop or a gravitational ghost loop. Notice that, while tadpoles with a zero momentum internal line ($n = 0$) are assumed to be exactly canceled by a suitable stabilization mechanism, the same type of diagrams with non zero n must be taken into account and they are crucial to recover gauge invariance for brane observables. Let us focus for instance on the mass correction δm_0^2 for a scalar on a brane at a generic y. Notice that for $y \neq 0, \pi R$ the brane is free to move, so branons must be kept. We get

$$\begin{aligned} \delta m_0^2 &= \sum_n \Big[2\cos^2(ny/R)\, F^{(n)}(\text{gravitons}) + 2\sin^2(ny/R)\, F^{(n)}(\text{branons}) + \\ &\qquad\qquad + \cos(2ny/R)\, F^{(n)}(\text{tadpole}) \Big] \\ &= m_0^2 \left(2G(M_D R) + \tilde{\Delta}_0(m_0, y, R, M_D, \bar{\mu}) \right). \end{aligned} \tag{3.19}$$

The contribution from the nth KK mode in the graviton diagrams of figure $1a$,b is exactly the same as in the torus, except for an overall factor $2\cos^2(ny/R)$ coming from the graviton wave function. The contribution $F^{(n)}$(branons) from diagrams in figure $1d$,e gets instead an overall y-dependent factor $2\sin^2(ny/R)$. The final result for δm_0^2 has the same structure of eq. (3.5), but now the gauge invariant piece $\tilde{\Delta}_0$ is a function of y. Finally, the tadpole contribution comes in the right way to cancel mode by mode the y-dependence in the gauge variant term G. This is consistent with the mass correction for the zero mode of a scalar propagating in the bulk, which has the same form as in the torus case (see eq. (3.6)), and it represents a non trivial check on the result. In the special case of a brane sitting at the fixed points $0, \pi R$ the branon contribution vanishes and gauge invariance of the pole mass is met just through tadpole diagrams.

As a final remark, we notice that because of the modified momentum conservation law in the orbifold, the zero mode of a bulk field mixes with its KK excitations at one loop level. The relevant diagrams are those in figures 1a,b,c with discrete momentum n in the internal loop and 0, $2n$ in the external legs. This effect however is relevant only at order κ^4 and can be safely neglected.

4. Phenomenology

In the previous sections we have explained how to consistently compute quantum gravity corrections using an effective field theory (EFT). A possible physical application is the computation of graviton loop corrections to electroweak precision observables (EWPO) and to the anomalous moment of the muon in brane models with large extra dimensions and a TeV-scale D-dimensional Planck mass. Unfortunately our knowledge of the low energy effective theory of gravity only allows to reliably compute corrections of little phenomenological interest. Basically, the EFT allows to compute those contributions that are saturated in the infrared, i.e. at the scale of the relevant external momenta. For instance, the calculable corrections to EWPO go like $(M_Z/M_D)^{2+\delta}$ (or $(M_Z/M_D)^{2+\delta} \ln M_Z$) by simple dimensional analysis. These effects go to zero very quickly when M_D is raised, becoming negligible already for M_D below a TeV. On the other hand, the contributions from the region of large virtual loop momenta gives in principle a much larger effect. However, being saturated in the UV region, where we do not control the EFT, these contributions are not calculable. This problem already affects tree level virtual graviton effects.

We can however *estimate* graviton effects by introducing an explicit UV cutoff Λ. The corrections to EWPO will scale like $M_Z^2 \Lambda^\delta / M_D^{2+\delta}$. The unknown physical cutoff could perhaps be produced by string theory, or could be related to the inverse brane width or even to just the brane tension [30]. Since we do not know we must keep Λ as a phenomenological parameter and discuss its physical meaning and plausible value.

Virtual graviton corrections (even at tree level) cannot be computed from Einstein gravity as much as electroweak quantum corrections cannot be computed from Fermi theory. In the latter case the complete theory is known and perturbative: by comparing to the full theory one sees that correct estimates are obtained by cutting off power divergent four-fermion loops at a "small" scale $\Lambda \sim M_W \approx g G_{\rm F}^{-1/2}$ rather than at the larger $\Lambda \approx G_{\rm F}^{-1/2}$. At least at a qualitative level, the gravitational Λ can be given a similar physical meaning.

4.1 Strong vs. weak gravity: NDA estimates

Therefore we first identify the value Λ_S of Λ that corresponds to strongly coupled quantum gravity.[8] This can be done by adapting to our case the naive dimensional analysis (NDA) technique developed to estimate pion interactions below the QCD scale [31] (NDA has already been applied to brane models [32]). NDA allows to estimate the size of the effects from a strongly coupled theory up to coefficients of order 1 but including all the geometric dependence on powers of π. By applying NDA, we estimate

$$\Lambda_S^{2+\delta} \approx \pi^{2-\delta/2}\Gamma(2+\delta/2) M_D^{2+\delta}\,. \tag{4.1}$$

In the range of interesting δ, Λ_S is not much larger than M_D.

We first discuss the particular case $\Lambda = \Lambda_S$: diagrams with any number of graviton lines give comparable contributions, and NDA allows to estimate their size. Tree level exchange of gravitons generates the effective dimension 8 operator $\mathcal{T} \equiv T_{\mu\nu}^2 - T_{\mu\mu}^2/(\delta+2)$ [4, 5, 6]. Its coefficient in the effective lagrangian is divergent, and NDA estimates it to be $\approx \pi^2/\Lambda_S^4$. This operator is however not the most important in low energy phenomenology, because at loop level gravitons generate dimension 6 four fermion operators with coefficient $\approx \pi^2/\Lambda_S^2$. On the other hand the operator $W^a_{\mu\nu} B^{\mu\nu} H^\dagger \tau^a H$ is generated with coefficient $\approx g_2 g_1/\Lambda_S^2$ with no π^2 enhancement. This property is shared by other operators that require the exchange of virtual gauge bosons. This is because we are assuming that the weak gauge couplings remain weak up to the cutoff.

By drawing a few Feynman graphs one can see that tree level exchange of gravitons (and therefore the operator $\mathcal{T}$) does not affect precision observables at the Z-resonance. Moreover the four fermion operators induced by double graviton exchange are of neutral current type, so they do not directly affect μ decay and are therefore not constrained by high precision data. μ-decay is affected by one loop diagrams with a W and a graviton: their coefficient is only $\approx g_2^2/\Lambda_S^2$.

Indeed by a simple analysis one finds that all dimension six operators that affect EWPO have a coefficient $\approx g_2^2/\Lambda_S^2 \approx 1/\Lambda_S^2$. As shown in [33] EWPO set a bound $\Lambda_S > (5 \div 10)\,\text{TeV}$ on a generic set of dimension 6 operators that conserve baryon, lepton and flavor numbers and CP. This bound seems rather strong when compared to the sensitivity to direct graviton emission expected at the next colliders. Furthermore since $m_{\rm top} \approx 175\,\text{GeV}$ a real solution of the hierarchy problem should cutoff the quadratically divergent top correction to the Higgs mass at a much lower value of

[8] In the context of string theory this corresponds to a situation where the string coupling is essentially at the self dual point.

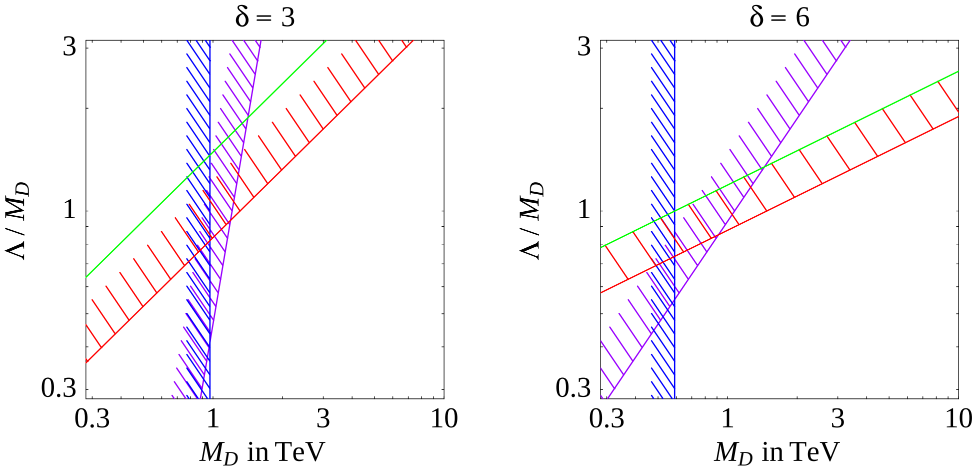

Figure 4: Compilation of collider bounds on graviton phenomenology in the plane $(M_D, \Lambda/M_D)$ for $\delta = 3$ (left) and $\delta = 6$ (right).

$\Lambda \approx 300\,\mathrm{GeV}$. Our assumption $\Lambda = \Lambda_S$ corresponds however to one of the most constrained scenarios: LEP data strongly disfavor new strongly coupled physics in the electroweak sector. The situation becomes worse if we assume that also the gauge couplings get strong at $\Lambda = \Lambda_S$. In order to obtain a more acceptable phenomenology one can assume that the UV cut-off Λ happens to be smaller than Λ_S, so that gravity does not become strong and dominant graviton corrections to EWPO are dominated by one loop diagrams (presumably a complete theory will not contain only gravitons). In the next subsections we 'compute' the graviton corrections to the electroweak observables (expressed in terms of the $\epsilon_1, \epsilon_2, \epsilon_3$ parameters [12]) and to the anomalous magnetic moment of the muon. In agreement with NDA estimates, the final result is of the form

$$\delta\epsilon_i \approx \frac{M_Z^2}{\Lambda_S^2}\left(\frac{\Lambda}{\Lambda_S}\right)^{\delta}, \qquad \delta a_\mu \approx \frac{m_\mu^2}{\Lambda_S^2}\left(\frac{\Lambda}{\Lambda_S}\right)^{\delta}, \tag{4.2}$$

where the factors of order one depend on the choice of cutoff. Not knowing which is the physical cutoff, we use dimensional regularization: with this choice loop integrals do not give powers of Λ. However, since we are considering a higher dimensional theory, powers of Λ arise from divergent sums over the KK levels of the gravitons. A different choice of the cutoff would give different results.

In figure 4 we summarize the present situation of collider graviton phenomenology by collecting the various bounds in the plane $(M_D, \Lambda/M_D)$:

- The vertical bound comes from emission of real gravitons [4] at LEP2 and Tevatron [34]. It does not depend on Λ (as long as the energy of the collider is less than Λ) because it is the only bound on really computable effects.

- Virtual exchange of gravitons at tree level generates the operator $\mathcal{T}$. Its coefficient depends on the cutoff Λ so that it cannot be computed from the low energy

EFT (in the literature there exists a variety of estimates [4, 6, 29], freely dubbed "formalisms", and a corresponding variety of experimental bounds [34]). The coefficient can be estimated to be $\approx \pi^2\delta(\delta+2)\Lambda^{\delta-2}/2(\delta-2)\Lambda_S^{\delta+2}$. The experimental constraints [34] give the slightly oblique bound in figure 4.

- At one loop gravitons affect precision observables and a_μ in a way that again depends on the cutoff. The green line shows the values necessary to produce the observed anomaly in a_μ. The bound parallel to it comes from precision observables.

If the cutoff Λ is due to quantum gravity, Λ/M_D parameterizes how strongly coupled gravity is: this explains why virtual graviton effects give the strongest (weakest) bound when $\Lambda \gtrsim M_D$ ($\Lambda \lesssim M_D$). Strongly coupled gravity is obtained for $\Lambda/M_D \sim (1\div 4)$ if $\delta = 3$ and for $\Lambda/M_D \sim (1\div 2)$ if $\delta = 6$. EWPO bounds have been estimated in a conservative way, assuming a typical 0.1% error. We see that setting $\Lambda = M_D$ as assumed in many analyses is a significant but arbitrary restriction: Λ is a relevant free parameter. In the most generic case the cutoff could even be not universal, so that different corrections are cut off by different Λ. We repeat that bounds that depend on Λ can at best be considered as semi-quantitative.

The presence of a cut-off Λ can have an impact on the studies of graviton emission at future colliders. If Λ is smaller than $\sqrt{s}$, real graviton signals are suppressed (but some new physics should show up). On the other hand, if Λ/M_D is too big, real graviton signals ($\gamma+$ missing energy) are forbidden by precision tests or subdominant with respect to $\gamma+$ missing energy effects due to dimension six operators like $e\bar{e}\nu\bar{\nu}$ [4], generated by virtual gravitons at one loop with coefficient $\sim \pi^2\Lambda^{2\delta+2}/\Lambda_S^{2\delta+4}$. However there exists a range of M_D and Λ (not too small and not too large) where real graviton emission is the dominant discovery mode. For instance one can see this by considering the case of a e^+e^- collider at $\sqrt{s} = 1$ TeV [4].

Can the apparent excess $a_\mu^{\rm exp} - a_\mu^{\rm SM} = (4.3 \pm 1.6)\cdot 10^{-9}$ recently measured by [35] be produced by gravitons without conflicting with the EWPO bounds? In the SM, electroweak corrections have been clearly seen in the ϵ_i, but only affect a_μ at a level comparable to its present experimental error. The naive (and maybe correct) expectation is that even in the gravitational case the ϵ_i are a more significant probe than a_μ. However, taking into account that we can only perform estimates, it could not be impossible that the anomaly in a_μ [35] be produced by gravity without conflicting with the EWPO bounds, even if the physical cutoff has a 'universal' nature (for example if it is related to the size of the brane) as assumed in figure 4. If this is the case, improved measurements of the ϵ_i parameters should be able to find a positive signal.

4.2 Electroweak precision observables

As discussed in the previous sections, unphysically large corrections cancel out when correctly computing physical observables. Previous analyses have studied certain combinations of the vacuum polarizations of the vector bosons

$$\Pi^{ij}_{\mu\nu}(k^2) = -i\eta_{\mu\nu}\Pi^{ij}(k^2) + k_\mu k_\nu \text{ terms}, \qquad i,j=\{W,Z,\gamma\} \tag{4.3}$$

known as S,T,U parameters [11], often employed to parameterize new physics present only in the vector boson sector. However these are not physical observables because gravity does not couple only to vector bosons.[9] As found in [7], in the unitary gauge gravitons give corrections to such parameters that unphysically increases with increasing M_D.

Since graviton loops are flavour universal (and neglecting the bottom quark mass) gravitational corrections to the various EWPO can be condensed in three parameters that are usually chosen to be $\epsilon_1, \epsilon_2, \epsilon_3$. The corrections to the physical EWPO are obtained by combining in a non immediate but standard way [12] various form factors. Specializing the general expressions to the case of gravity, the ϵ parameters are given by

$$\begin{aligned}
\epsilon_1 &= 2\delta g - \frac{\delta G}{G} - \frac{\delta M_Z^2}{M_Z^2} - \Pi'_{ZZ}(M_Z^2) \\
\epsilon_2 &= 2c^2\delta g - \frac{\delta G}{G} - \frac{\delta M_W^2}{M_W^2} + s^2\frac{\delta\alpha}{\alpha} - c^2\Pi'_{ZZ}(M_Z^2) \\
\epsilon_3 &= 2c^2\delta g - c^2\frac{\delta\alpha}{\alpha} - c^2\Pi'_{ZZ}(M_Z^2)
\end{aligned} \tag{4.4}$$

where

- $\delta M_i^2 \equiv -\Pi_{ii}(M_i^2)$ are the correction to the pole mass of the vector bosons and $\Pi'(k^2) \equiv d\Pi(k^2)/dk^2$.

- $\delta\alpha = -\Pi_{\gamma\gamma}(0)$ is the correction to the electric charge;

- δg is the common correction to the vector and axial form factors (gravity respects parity) in the $Z_\mu f\bar{f}$ interactions of an on-shell Z boson

  $$-i\frac{e}{2sc}\bar{f}\gamma_\mu(g_V - \gamma_5 g_A)(1+\delta g)f$$

  excluding the contribution from the Z vacuum polarization.

- δG is the correction to the $\mu \to e\bar{\nu}_e\nu_\mu$ decay amplitude.

- $s^2 \equiv 1 - c^2 \equiv \left[1 - \sqrt{1 - 4\pi\alpha/\sqrt{2}GM_Z^2}\right]/2 \approx 0.2311$

[9] Various studies on different new physics scenarios use the S,T,U approximation outside its domain of applicability.

Although it would be straightforward to perform a complete analysis, we will only study the gravitational correction to the combination

$$\bar{\epsilon} \equiv \epsilon_1 - \epsilon_2 - \frac{s^2}{c^2}\epsilon_3 = \frac{\delta M_W^2}{M_W^2} - \frac{\delta M_Z^2}{M_Z^2} \tag{4.5}$$

chosen because it only involves the simplest-to-compute form factors. Physically, this observable amounts to testing the tree level SM prediction $M_W = cM_Z$ using the value of the weak angle given by the forward-backward asymmetries in $Z \to \ell^+\ell^-$ decays, not affected by graviton loop effects. The experimental value of $\bar{\epsilon}$ (obtained from a fit of LEP and SLD data) is $\bar{\epsilon} = (12.5 \pm 1)10^{-3}$ and agrees with the SM prediction (for a light higgs). The gravitational correction is given in appendix C in terms of Passarino-Veltman functions. Since the heaviest KK give the dominant effect, we can explicitly write the graviton effect in the limit $m_n \gg M_Z$ as

$$\delta\bar{\epsilon} \approx \sum_n \frac{s^2 M_Z^2}{\bar{M}_4^2(4\pi)^2}\left[\frac{40+25\delta}{6+3\delta}\left(\frac{1}{\epsilon} + \ln\frac{\bar{\mu}^2}{m_n^2}\right) + \frac{424+546\delta+137\delta^2}{18(2+\delta)^2}\right],$$

where we set $d = 4 - 2\epsilon$. We can estimate the graviton correction by keeping only the logarithmic term, setting $\bar{\mu} = \Lambda$ and cutting off the sum at $n < R\Lambda$. This gives

$$\delta\bar{\epsilon} \approx \frac{s^2 M_Z^2}{M_D^2}\left(\frac{\Lambda}{M_D}\right)^\delta \frac{5(8+5\delta)}{48\Gamma(2+\delta/2)\pi^{2-\delta/2}}\,. \tag{4.6}$$

The result has a strong dependence on Λ and the numerical coefficient is specific of the form of the cutoff that we have chosen to employ. We explicitly see that spurious IR divergences do not affect this physical observable.

Notice that $\bar{\epsilon}$ is suppressed by a power of s^2 but it is not affected by theoretical uncertainties in $\delta\alpha$. Therefore precision searches for M_D could be improved by a factor ~ 3 if, by producing $\sim 10^9$ Z bosons at an $e\bar{e}$ linear collider, the errors on M_W and on the effective weak angle extracted from the leptonic asymmetries could be reduced by a factor ~ 10.

4.3 Anomalous magnetic moment of the μ

Since the μ anomalous magnetic moment is zero at tree level, the reparametrization gauge dependence of the unit of mass does not affect the one loop gravitational correction to a_μ. Only few Feynman diagrams contribute. As noticed in [36], the $1/\epsilon$ poles cancel out when computing the loop integrals using dimensional regularization around $d = 4$. At leading order in m_μ we find, again using a sharp cut off for the sum over KK modes

$$\delta a_\mu = \frac{m_\mu^2}{M_D^2}\left(\frac{\Lambda}{M_D}\right)^\delta \frac{34+11\delta}{96\Gamma(2+\delta/2)\pi^{2-\delta/2}}\,. \tag{4.7}$$

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Apparently this result agrees with the one found by [10].[10] Again the result *strongly* depends on the value of the cutoff Λ. We cannot claim that it has the same sign as the apparent excess recently measured by [35]: we have employed dimensional regularization for loop integrals but other regularizations (e.g. dimensional reduction, Pauli-Villars,...) would give a different result. Unlike $\bar{\epsilon}$, δa_μ is a sum of contributions from graphs with different graviton interactions. One can obtain any sign for δa_μ e.g. by cutting off $\bar{\mu}\mu h$ and $\gamma\gamma h$ vertices with different form factors: δa_μ is finite but dominated by loop momenta around the cutoff. In particular one gets, at leading order in m_μ

$$\delta a_\mu = 0 \tag{4.8}$$

if the cutoff acts in the same way on both type of contributions. This is for example the case of a Pauli-Villars cutoff on the graviton. By working in the De Donder gauge (where the only dependence on the graviton mass comes from the $1/(k^2 - m_n^2)$ factor in the graviton propagator) and knowing that a_μ is dimensionless and finite, it is not difficult to realize that it is zero.

5. Conclusions

We discussed various subtleties that arise when computing quantum gravity 1-loop effects in models with large extra dimensions and matter confined to a brane. Our computations are based on an effective field theory (EFT) description of quantum gravity and of the brane. A sensible result is obtained after correctly identifying physical observables and after taking brane fluctuations into account. Graviton tadpoles are relevant for branes living in non-homogeneous spaces (like orbifolds). For branes living at orbifold fixed points consistency is met, as expected, even in the absence of brane fluctuations. In particular we explain in a geometric way why the units of length in 'longitudinal' and 'transverse' directions depend on the reparametrization gauge fixing procedure.

We regard these results as theoretically interesting, although the truly calculable effects in the EFT approach have a limited phenomenological relevance. The most relevant effects come from the region of large virtual momenta where the EFT description breaks down. This is why in the second part of the paper we have abandoned the strict EFT approach and modeled these UV effects by introducing a hard momentum cut-off Λ. This is the best that can be done, without having a fundamental theory that allows real computations. We stress however that our previous understanding of how to get gauge independent results is still important in this phenomenological approach. As an application we have studied virtual graviton

[10][10] separately computes the gauge-dependent 'graviton' and 'radion' contributions in the unitary gauge. We find a different result in both cases (the radion coupling used in [10] is valid only on shell), but this discrepancy luckily cancels out when summing the two contributions.

corrections to precision observables and to the muon anomalous magnetic moment, focusing on models with large extra dimensions. Even at tree level, virtual graviton effects are divergent and must be regulated. Virtual graviton effects in collider phenomenology have been so far studied assuming a particular value of Λ. However Λ is an important free parameter that — at least at an qualitative level — controls how strongly coupled gravity is. Depending on the value of Λ, one loop effects can give the dominant bound on low scale quantum gravity.

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A. Graviton propagators in the unitary gauge

We derive here the propagators for the physical fields in the unitary gauge. Expanding the metric $g_{MN} = \eta_{MN} + \kappa h_{MN}$ around the flat space solution we obtain the lagrangian in eq. (2.1). We recall that we decompose the D dimensional graviton h_{MN} as $h_{\mu\nu}$, $h_{\mu i}$ and h_{ij} (where μ, ν are 4-dimensional indices and i, j span the extra δ dimensions). In this appendix we fix for simplicity $d = 4$. Due to D-dimensional reparametrization invariance not all the components of these fields correspond to propagating degrees of freedom. The physical fields are the ones contained in the Riemann tensor $2R_{RMSN} = h_{(RN,MS)} - h_{(MN,RS)}$ (as in the electromagnetic case the physical fields are contained in the field strength tensor)

$$\begin{aligned} G_{\mu\nu} &= -2\partial_i\partial_j R_{i\mu j\nu} = h_{\mu\nu} - \partial_i\partial_{(\mu}h_{\nu)i} + \partial_\mu\partial_\nu\partial_i\partial_j h_{ij} \\ V_{\mu i} &= +2\partial_j\partial_n R_{j\mu in} = h_{\mu i} - \partial_i\partial_n h_{\mu n} - \partial_\mu\partial_j h_{ij} + \partial_\mu\partial_i\partial_j\partial_n h_{jn} \\ S_{ij} &= -2\partial_m\partial_n R_{imjn} = h_{ij} - \partial_n\partial_{(i}h_{j)n} + \partial_i\partial_j\partial_m\partial_n h_{mn}\,. \end{aligned} \tag{A.1}$$

For simplicity, the above equations are written assuming units such that $\partial_i\partial_i = 1$. These expressions can be written in a compact form by defining $Q_\mu \equiv \hat\partial_i h_{\mu i}$, $P \equiv \hat\partial_i\hat\partial_j h_{ij}$ and $P_i = \hat\partial_j h_{ij} - \hat\partial_i P$, where $\hat\partial_i \equiv \partial_i/\sqrt{\partial_j\partial_j}$. These considerations suggest to rewrite the lagrangian in terms of a new set of fields [4] $G_{\mu\nu}$, $V_{\mu i}$, S_{ij}, H, Q_μ, P_i, P related to h_{MN} by

$$\begin{aligned} h_{\mu\nu} &= G_{\mu\nu} - \frac{c}{4-1}\left(\eta_{\mu\nu} + \frac{\partial_\mu\partial_\nu}{\partial_i^2}\right)H + \partial_\mu Q_\nu + \partial_\nu Q_\mu - \partial_\mu\partial_\nu P \\ h_{ij} &= S_{ij} + \frac{c}{\delta-1}(\eta_{ij} - \hat\partial_i\hat\partial_j)H + \hat\partial_i P_j + \hat\partial_j P_i + \hat\partial_i\hat\partial_j P \\ h_{\mu i} &= V_{\mu i} + \partial_\mu P_i + \hat\partial_i Q_\mu \end{aligned} \tag{A.2}$$

and subject to the constraints

$$\partial_i V_{\mu i} = \partial_i S_{ij} = \partial_i P_i = 0\,, \qquad S_{ii} = 0\,.$$

We have introduced the field H in order to make S_{ij} traceless. By choosing $c^2 = 3(\delta-1)/(\delta+2)$, H is canonically normalized. The D-dimensional lagrangian becomes

$$\begin{aligned}\mathcal{L} = & -\frac{1}{2}H(\Box+\partial_k^2)H - \frac{1}{2}S^{ij}(\Box+\partial_k^2)S_{ij} - V^{\mu i}[(\Box+\partial_k^2)\eta_{\mu\nu} - \partial_\mu\partial_\nu]V_i^\nu + \\ & +\frac{1}{2}G_\mu^\mu(\Box+\partial_k^2)G_\nu^\nu - \frac{1}{4}G^{\mu\nu}(\Box+\partial_k^2)G_{\mu\nu} + G^{\mu\rho}\partial_\rho\partial_\sigma G_\mu^\sigma - G_\rho^\rho\partial_\mu\partial_\nu G^{\mu\nu}\,. \end{aligned} \tag{A.3}$$

As expected, it does not depend on Q_μ, P_i and P and there is no mixing between the fields $G_{\mu\nu}$, H, S_{ij}, $V_{\mu i}$. It is now trivial to perform a mode expansion: the extra dimensional Laplacian ∂_k^2 becomes a mass term. The propagators can be obtained by inverting the kinetic terms in eq. (A.3). It is useful to show explicitly how the 'graviton' $G_{\mu\nu}$ and the 'scalar' H combine to give a unitary-gauge propagator equal to the de-Donder propagator, up to longitudinal terms. For example, the $h_{\mu\nu}h_{\mu'\nu'}$ propagator is

$$\begin{aligned}\overbrace{h_{\mu\nu}^{(n)}h_{\mu'\nu'}^{(n')}} &= \overbrace{G_{\mu\nu}^{(n)}G_{\mu'\nu'}^{(n')}} + \frac{\delta-1}{3(\delta+2)}t_{\mu\nu}t_{\mu'\nu'}\overbrace{H^{(n)}H^{(n')}} \\ &= \frac{i\delta_{n,-n'}}{2(k^2-m_n^2)}\left(t_{\mu\mu'}t_{\nu\nu'} + t_{\mu\nu'}t_{\nu\mu'} - \frac{2}{2+\delta}t_{\mu\nu}t_{\mu'\nu'}\right). \end{aligned} \tag{A.4}$$

In particular we see that we cannot omit the 'scalar' contributions, if we want to obtain a gauge invariant result. It would be easy to include a small mass term for the $H^{(n)}$ fields, eventually generated by the unknown mechanism that stabilizes the size of the extra dimensions.

B. Graviton vertices

We define $g_{\mu\nu} \equiv \eta_{\mu\nu} + \kappa\, h_{\mu\nu}$, $g \equiv |\det g_{\mu\nu}|$ and give explicit expressions for the expansion up to second order in the graviton field $h_{\mu\nu}$ of

$$\begin{aligned}\sqrt{g} &= 1 + \kappa A^{\alpha\beta}h_{\alpha\beta} + \kappa^2 A'^{\alpha\beta\gamma\delta}h_{\alpha\beta}h_{\gamma\delta} + \cdots \\ \sqrt{g}g^{\mu\nu} &= 1 + \kappa B^{\mu\nu\alpha\beta}h_{\alpha\beta} + \kappa^2 B'^{\mu\nu\alpha\beta\gamma\delta}h_{\alpha\beta}h_{\gamma\delta} + \cdots \\ \sqrt{g}g^{\mu\rho}g^{\nu\sigma} &= 1 + \kappa C^{\mu\nu\rho\sigma\alpha\beta}h_{\alpha\beta} + \kappa^2 C'^{\mu\nu\rho\sigma\alpha\beta\gamma\delta}h_{\alpha\beta}h_{\gamma\delta} + \cdots\,. \end{aligned} \tag{B.1}$$

In the vierbein formalism the spin connection is given by

$$\omega_a{}^{cd} = e_a^\mu \omega_\mu{}^{cd} = e_a^\mu\left(e^{\nu c}\partial_{[\mu}e^d_{\nu]} - e^{\nu d}\partial_{[\mu}e^c_{\nu]}\right) - e^{\rho c}e^{\sigma d}\partial_{[\rho}e^m_{\sigma]}\eta_{ma}\,, \tag{B.2}$$

We expand around the flat background δ_μ^a, $e_\mu^a = \delta_\mu^a + \kappa\, b_\mu^a$. As discussed in section 2.4, the gauge choice $b_{[\mu\,\nu]} = 0$ allows to express $b_{\mu\nu}$ in terms of $h_{\mu\nu}$

$$b_{\mu\nu} = \frac{1}{2}h_{\mu\nu} - \frac{\kappa}{8}h_\mu^\alpha h_{\alpha\nu} + O(\kappa^2)\,. \tag{B.3}$$

Using (B.2), (B.3) and

$$e^{\mu}_{a} = \delta^{\mu}_{a} - \kappa b^{\mu}_{a} + \kappa^2 b^{\mu}_{\alpha} b^{\alpha}_{a} + O(\kappa^3) \tag{B.4}$$

one can find the gravitational couplings for fermions.

With these expressions it is straightforward to find the graviton vertices arising from lagrangians like

$$\begin{aligned}\mathcal{L} = \sqrt{g}\Big[& g^{\mu\nu}\frac{(\partial_\mu\phi)(\partial_\nu\phi)}{2} - \frac{m_\phi^2}{2}\phi^2 - \frac{1}{4}g^{\mu\rho}g^{\nu\sigma}F_{\mu\nu}F_{\rho\sigma} - \frac{m_A^2}{2}g^{\mu\nu}A_\mu A_\nu + \\ &+ \frac{i}{2}\left(\overline{\psi}e^{\mu}_{a}\gamma^a D_\mu\psi - D^{\dagger}_{\mu}\overline{\psi}e^{\mu}_{a}\gamma^a\psi\right)\Big],\end{aligned} \tag{B.5}$$

where $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$ and $D_\mu\psi = \partial_\mu\psi + \frac{1}{2}\omega_\mu{}^{ab}\gamma_{ab}\psi$, $\gamma^{ab} = \frac{1}{4}[\gamma^a,\gamma^b]$. The expansion in powers of h is easily obtained from

$$\begin{aligned} g^{\mu\nu} &= \eta^{\mu\rho}[\delta^{\nu}_{\rho} - \kappa h^{\nu}_{\rho} + \kappa^2 (hh)^{\nu}_{\rho} - \kappa^3 (hhh)^{\nu}_{\rho} + \cdots] \\ \sqrt{g} &= 1 + \kappa\frac{\mathrm{Tr}\, h}{2} + \kappa^2\left[\frac{\mathrm{Tr}^2 h}{8} - \frac{\mathrm{Tr}\, h^2}{4}\right] + \kappa^3\left[\frac{\mathrm{Tr}^3 h}{48} - \frac{\mathrm{Tr}\, h\,\mathrm{Tr}\, h^2}{8} + \frac{\mathrm{Tr}\, h^3}{6}\right] + \cdots . \end{aligned} \tag{B.6}$$

Therefore

$$\begin{aligned} A^{\alpha\beta} &= \frac{1}{2}\eta^{\alpha\beta} \\ B^{\mu\nu\alpha\beta} &= 4A'^{\mu\nu\alpha\beta} = \frac{1}{2}(\eta^{\mu\nu}\eta^{\rho\sigma} - \eta^{\mu\rho}\eta^{\nu\sigma} - \eta^{\mu\sigma}\eta^{\nu\rho}) \\ B'^{\mu\nu\alpha\beta\gamma\delta} &= \frac{1}{2}\eta^{\mu\nu}B^{\rho\sigma\alpha\beta} - \left(\eta^{\mu\rho}B^{\alpha\beta\nu\sigma} + \eta^{\nu\rho}B^{\alpha\beta\mu\sigma}\right) \\ C^{\mu\nu\rho\sigma\alpha\beta} &= \left[\frac{1}{2}\eta^{\mu\rho}\eta^{\nu\sigma}\eta^{\alpha\beta} - \eta^{\mu\alpha}\eta^{\beta\rho}\eta^{\nu\sigma} - \eta^{\nu\alpha}\eta^{\beta\sigma}\eta^{\mu\rho}\right] \\ C'^{\mu\nu\rho\sigma\alpha\beta\gamma\delta} &= \left[\frac{1}{4}B^{\alpha\beta\gamma\delta}\eta^{\mu\rho}\eta^{\nu\sigma} + 2\eta^{\gamma\mu}\eta^{\alpha\rho}\eta^{\delta\beta}\eta^{\nu\sigma} - \eta^{\gamma\delta}\eta^{\alpha\mu}\eta^{\beta\rho}\eta^{\nu\sigma} + \eta^{\alpha\mu}\eta^{\beta\rho}\eta^{\gamma\nu}\eta^{\delta\sigma}\right] \end{aligned} \tag{B.7}$$

These expressions are valid in any number of dimensions. Brane fluctuations can be incorporated in $h_{\mu\nu}$, as discussed in eq. (2.12).

To compute the corrections to the graviton propagator and to the graviton vertex it is necessary to have the 3 and 4 graviton interactions. They can be easily derived by expanding the Einstein-Hilbert lagrangian in powers of the graviton field using e.g. Mathematica [28]. For this reason we do not write explicitlt the long expressions for such vertices.

C. Results

In this appendix we collect the explicit results for the corrections to the propagator of a spin 0,1 particle confined on a brane with dimension d living in $\mathbb{R}^d \times T^\delta$.

As discussed in section 3, generically the correction to a physical quantity $\mathcal{O}$ with canonical dimension $d_{\mathcal{O}}$ has the form

$$\frac{\delta\mathcal{O}}{\mathcal{O}} = d_{\mathcal{O}} G(M_D R) + \Delta(\mathcal{O}, R, M_D, \mu)\,, \tag{C.1}$$

where the gauge dependence is encoded in the function G. The splitting in a 'gauge-dependent' and 'gauge-independent' part is ambiguous unless a reference gauge is chosen in which by definition one sets $G_{\rm ref} = 0$. We choose in the de Donder gauge $G_{\rm deDonder} = 0$. All the results for physical quantities are computed in this gauge. The results are expressed in terms of Passarino-Veltman functions A_0, $B_{0,1}$, defined in appendix D.

For the pole mass correction for a scalar ($s = 0$) and a massive vector ($s = 1$) on the brane we find

$$\begin{aligned}
G_{\rm unitary} &= -\sum_n A_0(m_n^2)\frac{[d^2+(\delta-3)\delta+d(2\delta-1)]}{16\pi^2\bar{M}_d^{d-2}d(d+\delta-2)}\\
\Delta_0 &= \frac{1}{64\pi^2\bar{M}_d^{d-2}d(d+\delta-2)}\sum_n\Big\{2d(2-d)(\delta-4)A_0(m_0^2)+f_1(d,\delta)\times\\
&\qquad\times A_0(m_n^2)+8d[m_n^2(d-2)-2m_0^2(d+\delta-3)]\times\\
&\qquad\times B_0(m_0^2,m_n^2,m_0^2)+2d\delta(d-2)m_n^2\times\\
&\qquad\times B_1(m_0^2,m_n^2,m_0^2)\Big\}\\
\Delta_1 &= \frac{1}{64\pi^2\bar{M}_d^{d-2}d(d-1)(d+\delta-2)}\sum_n\Big\{f_2(d,\delta)A_0(m_1^2)+(d-1)f_1(d,\delta)\times\\
&\qquad\times A_0(m_n^2)-8d[2m_1^2(d-1)(d+\delta-3)+\\
&\qquad+m_n^2(d-2)(d+2\delta+1)]\times\\
&\qquad\times B_0(m_1^2,m_n^2,m_1^2)+2d(d-2)\times\\
&\qquad\times[2d^2+3d(\delta-2)-7\delta]m_n^2\times\\
&\qquad\times B_1(m_1^2,m_n^2,m_1^2)\Big\}
\end{aligned} \tag{C.2}$$

with

$$\begin{aligned}
f_1(d,\delta) &= 4[2d^3-d(\delta-6)+2d^2(\delta-3)+2\delta(\delta-2)]\\
f_2(d,\delta) &= 2d(2-d)[4+2d^2+\delta+d(3\delta-2)]\,.
\end{aligned} \tag{C.3}$$

We computed also the graviton correction to the photon propagator, verifying that it is transverse if one uses the simple gauge fixing of eq. (2.20). If instead the gauge fixing function contains the graviton field, in general transversality will be lost, due to a modification of the related Ward identity (of course this does not mean that the photon acquires a mass). As the simple QED case, Ward identities imply that the photon self energy at zero momentum gives the correction to the electric charge. We find

$$\Delta_e = -\sum_n A_0(m_n^2) \frac{(d-4)[d^3 + d^2(\delta-5) + d(8-3\delta) + 2\delta(\delta+2)]}{32\pi^2 \bar{M}_d^{d-2} d(d+\delta-2)} . \tag{C.4}$$

For comparison with the existing literature, we also write the expression of the scalar self-energy at zero momentum $\Sigma(0)$. As discussed in the text, for this unphysical quantity the gauge dependent part is non-universal. In the de Donder and in the unitary gauge we find

$$\begin{aligned} \Sigma(0)_{\rm deDonder} &= \frac{m_0^2}{32\pi^2 \bar{M}_d^{d-2}(d+\delta-2)} \sum_n \frac{1}{(m_0^2 - m_n^2)} \times \\ &\quad \times \Big\{ [m_0^2 g_1(d,\delta) + m_n^2 g_2(d,\delta)] A_0(m_n^2) + 2d(\delta-2) m_0^2 A_0(m_0^2) \Big\} \end{aligned} \tag{C.5}$$

$$\begin{aligned} \Sigma(0)_{\rm unitary} &= \frac{m_0^2}{32\pi^2 \bar{M}_d^{d-2}(d+\delta-2)} \sum_n \frac{1}{m_n^4(m_0^2 - m_n^2)} \times \\ &\quad \times \Big\{ 2m_0^2 [m_n^4 d(\delta-2) - 2m_n^2 m_0^2(\delta-2) + m_0^4(d+\delta-3)] \times \\ &\qquad \times A_0(m_0^2) + (d-1) m_n^4 [m_0^2(d^2 + d(\delta-2) - 2) - \\ &\qquad - m_n^2(d-2)(d+\delta)] A_0(m_n^2) \Big\} \end{aligned} \tag{C.6}$$

with

$$\begin{aligned} g_1(d,\delta) &= d^2(d-1) - 4d + \delta(4+d)(d-1) + 2\delta^2 \\ g_2(d,\delta) &= 4\delta - (d+\delta)[2\delta + d(d-1)] \end{aligned} \tag{C.7}$$

From these expressions it is clear that $1/m_n^4$ terms found in [7, 8, 9] are an artifact of the unitary gauge and have no physical meaning.

D. Regularized sums and integrals

The results of our 1-loop computations can be expressed as the sum over KK modes of basic Passarino-Veltman functions [37]. Generically these expressions are divergent and need to be regulated. After doing that one can extract the calculable finite parts that are determined by the EFT [13]. These are terms that either depend on the radius R or depend non-analytically on the kinematic variables. In this appendix we focus for illustration on these calculable terms and disregard the uncalculable UV saturated contribution, which were the subject of our phenomenological discussion.

The main point is to regularize the integral and the series consistently; we choose for this the dimensional technique, extending the physical dimension of the extra space and of the brane δ, d, to generic values

$$\bar{\delta} = \delta - \epsilon; \qquad \bar{d} = d - \epsilon \tag{D.1}$$

rescaling the Planck mass in the lagrangian as $M_D^{d+\delta-2} \to M_D^{d+\delta-2}\mu^{\bar{d}+\bar{\delta}-d-\delta}$ and consequently

$$\frac{1}{\bar{M}_d^{d-2}} \equiv \frac{1}{M_D^{D-2}R^\delta} \to \frac{\mu^{2\epsilon}}{M_D^{D-2}R^{\bar{\delta}}} \tag{D.2}$$

and taking the limit $\epsilon \to 0$ at the end. Defining the Passarino-Veltman functions A_0, $B_{0,1}$

$$\begin{aligned} A_0(m^2) &= -i(4\pi)^2 \int \frac{d^dq}{(2\pi)^d}\frac{1}{q^2-m^2} \\ B_0(p^2,M^2,m^2) &= -i(4\pi)^2 \int \frac{d^dq}{(2\pi)^d}\frac{1}{(q^2-M^2)[(q+p)^2-m^2]} \\ B_1(p^2,M^2,m^2) &= \frac{-i(4\pi)^2}{p^2} \int \frac{d^dq}{(2\pi)^d}\frac{p\cdot q}{(q^2-M^2)[(q+p)^2-m^2]} \end{aligned} \tag{D.3}$$

the following expressions are sufficient to compute the gravitational corrections in appendix C

$$\sum_{n\in\mathbb{Z}^\delta} A_0(m_n^2) \qquad \sum_{n\in\mathbb{Z}^\delta} m_n^{2\alpha} B_{0,1}(m^2,m_n^2,m^2) \qquad\qquad \alpha = 0,1 \tag{D.4}$$

together with the series

$$\mathcal{I}_\alpha = \sum_{n\in\mathbb{Z}^\delta} \frac{1}{n^{2\alpha}}\,. \tag{D.5}$$

To illustrate the technique we compute explicitly $\sum_n B_0(m^2,m_n^2,m^2)$. First of all we introduce a Feynman parameter x, rescale the integration variable $q \to q/R$ and isolate the zero point in the series

$$\begin{aligned} \sum_{n\in\mathbb{Z}^{\bar{\delta}}} B_0(m^2,m_n^2,m^2) = -\frac{4i}{(2\pi)^{\bar{d}-2}} \int_0^1 dx \Bigg[&\sum_{n\in\mathbb{Z}^{\bar{\delta}}-\{0\}} \int d^{\bar{d}}q \frac{(\sqrt{x}/R)^{\bar{d}-4}}{[q^2-n^2-a^2(x)]^2} + \\ &+ \int d^{\bar{d}}q \frac{1}{[q^2-(1-x)^2m^2]^2} \Bigg], \end{aligned} \tag{D.6}$$

where $a^2(x) = R^2m^2(1-x)^2/x$. Then we Wick-rotate and evaluate the first term using the Schwinger's proper time method

$$\begin{aligned} \sum_{n\in\mathbb{Z}^{\bar{\delta}}-\{0\}} \int d^{\bar{d}}q \frac{1}{[q^2+n^2+a^2(x)]^2} &= \sum_{n\in\mathbb{Z}^{\bar{\delta}}-\{0\}} \int_0^\infty dt \int d^{\bar{d}}q \frac{t}{\Gamma(2)} e^{-t(q^2+n^2+a^2)} \\ &= \pi^2 \int_0^\infty dy [\mathcal{B}^{\bar{\delta}}(y)-1] e^{-y\pi a^2} y^{1-\bar{d}/2}\,, \end{aligned} \tag{D.7}$$

where we have performed the gaussian integral over q and introduced the special function

$$\mathcal{B}(s) \equiv \sum_{n=-\infty}^{\infty} e^{-\pi n^2 s}\,. \tag{D.8}$$

The integral in eq. (D.7) converges at $y \to \infty$ thanks to the exponential behavior of the $\mathcal{B}$ function, but it diverges at $y \to 0$. To extract the singularity it is useful the property

$$\mathcal{B}(s) = s^{-1/2}\mathcal{B}(\frac{1}{s}) \tag{D.9}$$

which is easily derived from the Poisson formula. Using eq. (D.9) we can split the integration interval and change variable $y \to 1/y$ in the first integral

$$\begin{aligned}\mathcal{I} &\equiv \Big(\int_0^1 + \int_1^\infty\Big) dy\; y^{1-\bar{d}/2} e^{-y\pi a^2} [\mathcal{B}^{\bar{\delta}}(y) - 1] \\ &= \int_1^\infty dy\; [\mathcal{B}^{\bar{\delta}}(y) - 1](y^{1-\bar{d}/2} e^{-y\pi a^2} + y^{(\bar{d}+\bar{\delta}-3)/2} e^{-\pi a^2/y}) + \\ &\quad + \int_1^\infty dy\; e^{-\pi a^2/y} y^{\bar{d}/2-3}(y^{\bar{\delta}/2} - 1)\end{aligned} \tag{D.10}$$

Again, the first integral is convergent, while the second term must be (dimensionally) regularized. Because $a(x) \geq 0$ in $x \in [0,1]$ and noting that for $\beta \geq 0$

$$\int_1^\infty dy\; y^\alpha e^{-\beta/y} = \beta^{\alpha+1}\Gamma(-\alpha-1) - \int_1^\infty dy\; y^{\alpha+2} e^{-\beta y} \tag{D.11}$$

we can isolate the divergent piece in the Γ function through an analytical continuation in the physical region $\bar{d} + \bar{\delta} \geq 0$

$$\begin{aligned}\mathcal{I} = &\int_1^\infty dy\; [\mathcal{B}^{\bar{\delta}}(y) - 1]\left(y^{1-\bar{d}/2} e^{-y\pi a(x)^2} + y^{(\bar{d}+\bar{\delta})/2-3} e^{-\pi a(x)^2/y}\right) + \\ &+ \int_1^\infty dy y^{1-\bar{d}/2} e^{-\pi y a(x)^2}(1 - y^{-\bar{\delta}/2}) + \left[\pi R^2 m^2 \frac{(1-x)^2}{x}\right]^{(\bar{d}+\bar{\delta})/2-2} \Gamma\left(2 - \frac{\bar{d}+\bar{\delta}}{2}\right) - \\ &- \left[\pi R^2 m^2 \frac{(1-x)^2}{x}\right]^{\bar{d}/2-2} \Gamma\left(2 - \frac{\bar{d}}{2}\right).\end{aligned} \tag{D.12}$$

It's not difficult to verify that the last (divergent) term is exactly canceled by the the zero mode contribution of the series in eq. (D.6). Putting together the remaining terms we obtain

$$\begin{aligned}\sum_{n\in\mathbb{Z}^{\bar{\delta}}} B_0(m^2, m_n^2, m^2) = &\frac{1}{(2\pi R)^{\bar{d}-4}} \int_0^1 dx\; x^{\bar{d}/2-2} \times \\ &\times \Big\{ f_1(mR, x, \bar{d}, \bar{\delta}) + \left[\pi R^2 m^2 \frac{(1-x)^2}{x}\right]^{(\bar{d}+\bar{\delta})/2-2} \Gamma(2 - \frac{\bar{d}+\bar{\delta}}{2}) \Big\},\end{aligned} \tag{D.13}$$

where we have defined

$$\begin{aligned}f_1(mR, x, d, \delta) = \int_1^\infty dy\; \Big\{&[\mathcal{B}^\delta(y) - 1](y^{1-d/2} e^{-y\pi a^2(x)} + y^{(d+\delta)/2-3} e^{-\pi a^2(x)/y}) + \\ &+ e^{-y\pi a^2(x)} y^{1-d/2}(1 - y^{-\delta/2})\Big\}.\end{aligned} \tag{D.14}$$

The Γ function in eq. (D.13) has poles for negative integer arguments and before taking the limit $\epsilon \to 0$ we must distinguish the two cases of even and odd $(d+\delta)$. If $(d+\delta)$ is *even*, a logarithmic term appears

$$
\begin{aligned}
F(\bar{d},\bar{\delta})\frac{\mu^{2\epsilon}}{R^{\bar{\delta}}}\sum_{n\in\mathbb{Z}^{\bar{\delta}}} B_0(m^2,m_n^2,m^2) = \frac{F(d,\delta)}{(2\pi)^{d-4}}\int dx x^{d/2-2}\Big\{ & \frac{1}{R^{d+\delta-4}} f_1(mR,x,d,\delta) + \\
& + \frac{m^{d+\delta-4}}{\Gamma\left((d+\delta)/2-1\right)}\left[-\frac{\pi(1-x)^2}{x}\right]^{(d+\delta)/2-2} \times \\
& \times\Big(\frac{1}{\epsilon} + \log\frac{\mu^2}{m^2} + \log\frac{2\sqrt{x}}{(1-x)^2} - \gamma_E + \\
& + \frac{1}{F(d,\delta)}\frac{dF}{d\epsilon}\Big|_{\epsilon=0}\Big)\Big\}, \qquad \text{(D.15)}
\end{aligned}
$$

where $F(d,\delta)$ is a generic function of d,δ which multiplies the integral in the physical amplitudes and the factor $1/R^{\bar{\delta}}$ comes from the graviton wave function normalization. By subtracting just the pole $1/\epsilon$ we get the loop correction in the MS scheme. Notice that the finite part contains a scheme independent $\log m$ term. When $(d+\delta)$ is *odd* we find instead a finite result

$$
\begin{aligned}
F(\bar{d},\bar{\delta})\frac{\mu^{2\epsilon}}{R^{\bar{\delta}}}\sum_{n\in\mathbb{Z}^{\bar{\delta}}} B_0(m^2,m_n^2,m^2) = \frac{F(d,\delta)}{(2\pi)^{d-4}}\int dx x^{d/2-2}\Big\{ & \frac{1}{R^{d+\delta-4}} f_1(mR,x,d,\delta) + \\
& + m^{d+\delta-4}\Gamma\left(2-\frac{d+\delta}{2}\right)\left[\frac{\pi(1-x)^2}{x}\right]^{(d+\delta)/2-2}\Big\}.
\end{aligned}
$$

Although there is no logarithm, the term $m^{d+\delta-4}$ represents a scheme independent finite effects as it depends non analytically on the lagrangian parameter m^2. The same technique can be used to compute the finite part of the other integrals in eq. (D.4) and the series in eq. (D.5); here we collect only the final results omitting the derivation (for $d>2$, $\alpha<\delta/2$)

$$
\begin{aligned}
\sum_{n\in\mathbb{Z}^{\bar{\delta}}} A_0(m_n^2) &= \frac{-4\pi}{(2\pi R)^{d-2}}\left[\int_1^\infty dy\,[\mathcal{B}^\delta(y)-1]y^{d/2}(1+y^{\delta/2-2}) + \frac{2}{d-2} - \frac{2}{d+\delta-2}\right] \\
\mathcal{I}_\alpha &= \frac{\pi^\alpha}{\Gamma(\alpha)}\left[\int_1^\infty dy\,[\mathcal{B}^\delta(y)-1](y^{\alpha-1}+y^{\delta/2-1-\alpha}) - \frac{1}{\alpha} - \frac{1}{\delta/2-\alpha}\right] \qquad \text{(D.16)}
\end{aligned}
$$

Notice that the sum of the $A_0(m_n^2)$ function has no $1/\epsilon$ pole. The special cases $\mathcal{I}_0, \mathcal{I}_1$ are needed respectively to evaluate terms $\sum A_0(m_{0,1}^2)$ in the results of the previous section and the series of eq. (3.16)

$$
\begin{aligned}
\mathcal{I}_0 &= -1 \\
\mathcal{I}_1 &= \pi\left[\int_1^\infty dy[\mathcal{B}^\delta(y)-1]\left(1+y^{\delta/2-2}\right) - \frac{\delta}{\delta-2}\right] \qquad \text{(D.17)}
\end{aligned}
$$

Finally, for $(d+\delta)$ *even*

$$
\begin{aligned}
F(\bar{d},\bar{\delta})\frac{\mu^{2\epsilon}}{R^{\bar{\delta}}}\sum_{n\in\mathbb{Z}^{\bar{\delta}}} m_n^2 B_i(m^2,m_n^2,m^2) = & -\frac{\delta F(d,\delta)}{(2\pi)^{d-3}}\int dx x^{d/2-2}u_i(x)\times \\
& \times\Big\{\frac{1}{R^{d+\delta-2}}f_2(mR,x,d,\delta)+\frac{m^{d+\delta-2}}{\Gamma\left((d+\delta)/2\right)}\times \\
& \times\left[-\frac{\pi(1-x)^2}{x}\right]^{(d+\delta)/2-1}\Big(\frac{1}{\epsilon}+\log\frac{\mu^2}{m^2}+ \\
& +\log\frac{2\sqrt{x}}{(1-x)^2}-\gamma_E+\frac{1}{F(d,\delta)}\frac{dF}{d\epsilon}\Big|_{\epsilon=0}-\frac{1}{\delta}\Big)\Big\}
\end{aligned}
$$

while for $(d+\delta)$ *odd*

$$
\begin{aligned}
F(\bar{d},\bar{\delta})\frac{\mu^{2\epsilon}}{R^{\bar{\delta}}}\sum_{n\in\mathbb{Z}^{\bar{\delta}}} m_n^2 B_i(m^2,m_n^2,m^2) = & -\frac{\delta F(d,\delta)}{(2\pi)^{d-3}}\int dx x^{d/2-2}u_i(x)\times \\
& \times\Big\{\frac{1}{R^{d+\delta-2}}f_2(mR,x,d,\delta)+m^{d+\delta-2}\times \\
& \times\Gamma\left(1-\frac{d+\delta}{2}\right)\left[\frac{\pi(1-x)^2}{x}\right]^{(d+\delta)/2-1}\Big\}
\end{aligned}
$$

where $i=0,1$ and $u_0(x)=1$, $u_1(x)=(x-1)$ and we have defined

$$
\begin{aligned}
f_2(mR,x,d,\delta)= & \int_1^\infty dy\, e^{-y\pi a^2(x)}[2B'(y)B^{\delta-1}(y)+y^{-(d+\delta)/2}]- \\
& -\int_1^\infty dy\, e^{-\pi a^2(x)/y}y^{(d+\delta)/2-2}[2yB'(y)B^{\delta-1}(y)+(B(y)-1)B^{\delta-1}(y)]\,.
\end{aligned}
$$

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